Mixed Hypergraphs for Linear-Time Construction of Denser Hashing-Based Data Structures

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Ilmenau University of Technology

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x_i:	Kr	ak	on	oš	
v_i:	2	1	5	5	$\in(\mathbb{Z}_6,+)$

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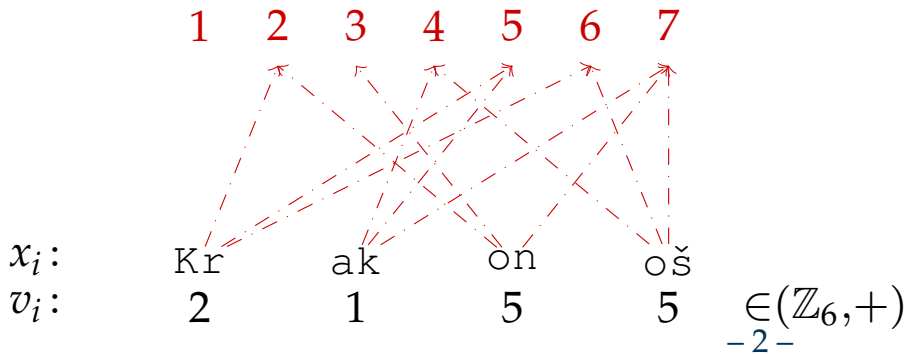

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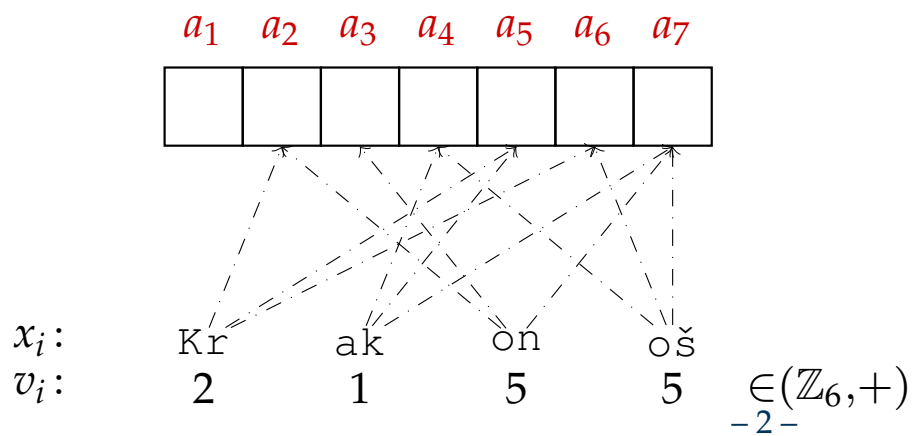

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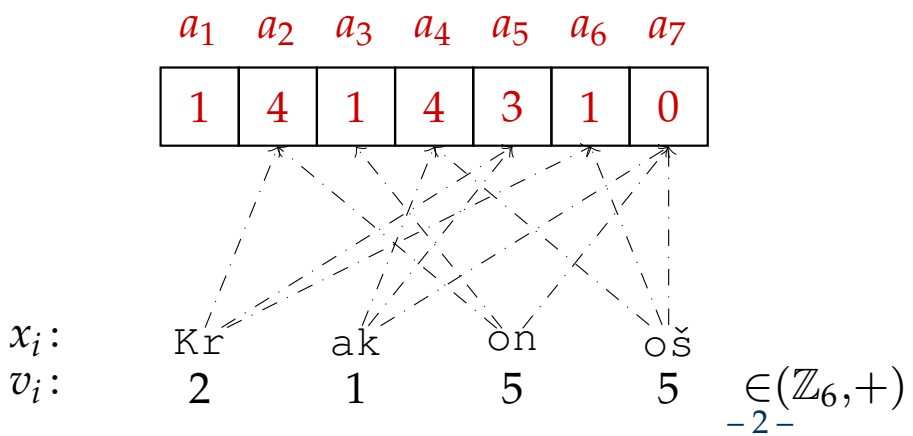

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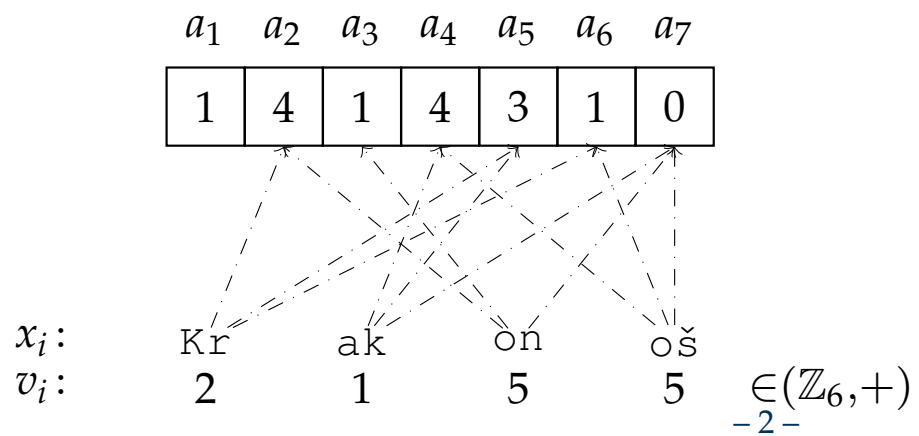
- find $a_1, a_2, \ldots, a_n \in V$ s.t. $\bigoplus_j a_{h_j(x_i)} = v_i$ for $i = 1, 2, \ldots, m$
- store $a_1, a_2, \ldots, a_n$ and $h_1, h_2, \ldots, h_k$;



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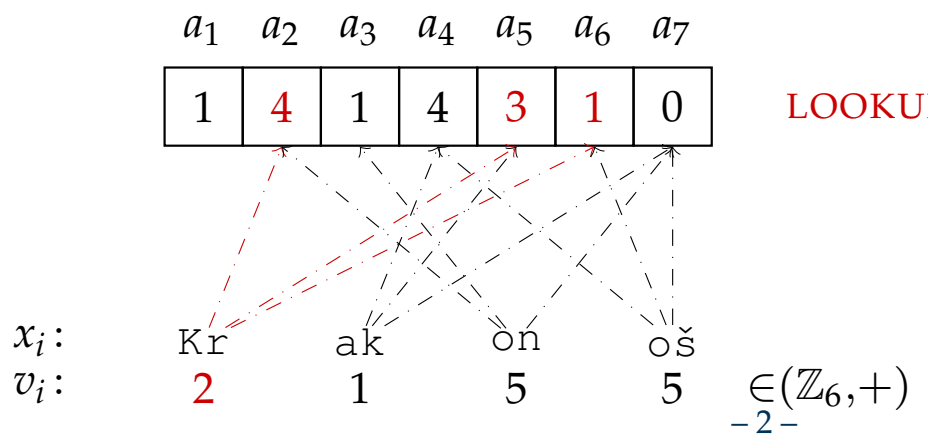
- store $a_1, a_2, \ldots, a_n$ and $h_1, h_2, \ldots, h_k$; LOOKUP$(x) := \bigoplus_j a_{h_j(x)}$



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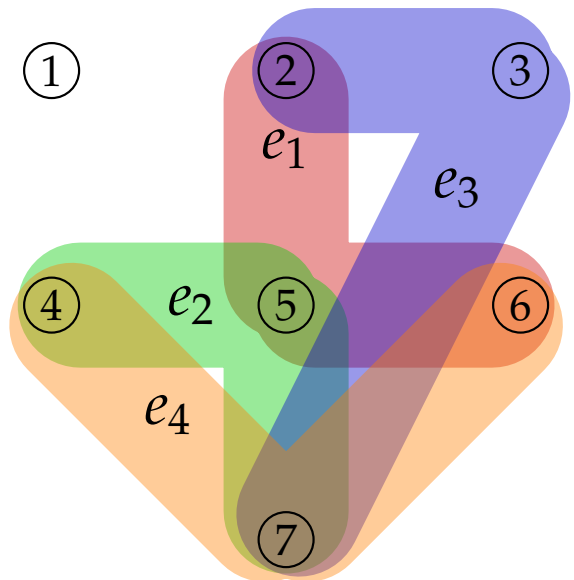

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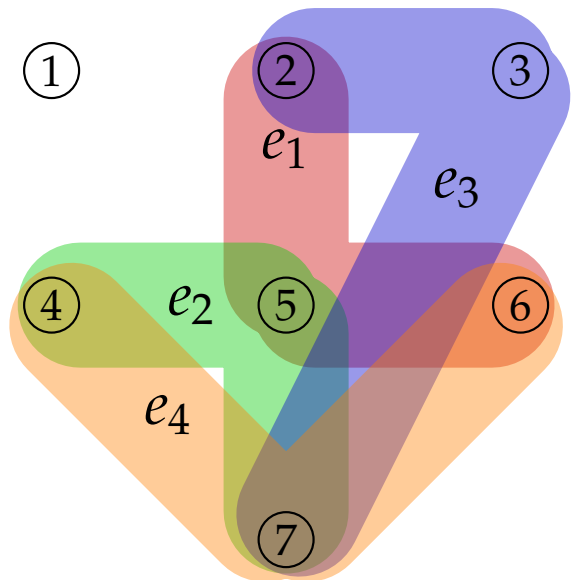

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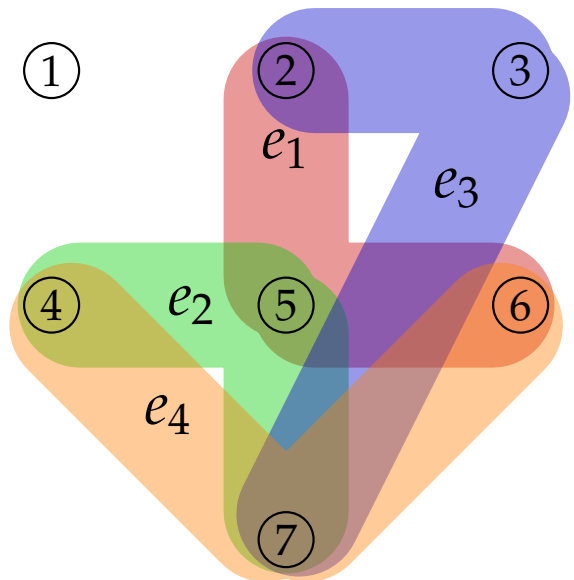

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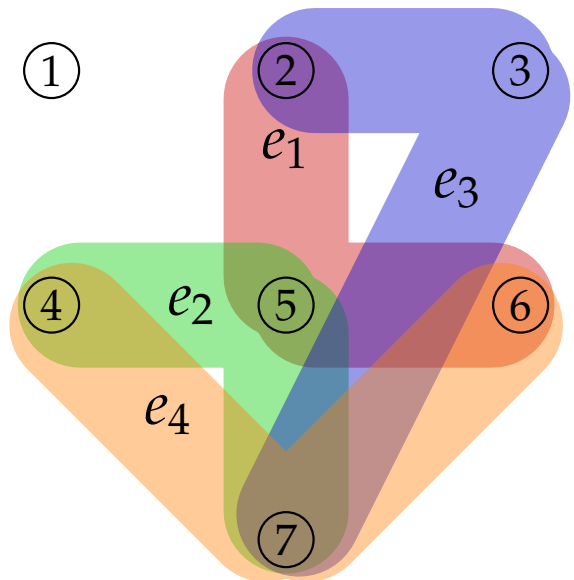

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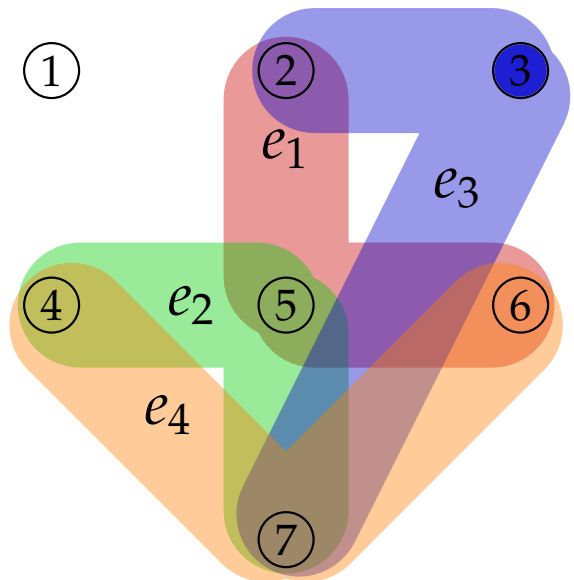

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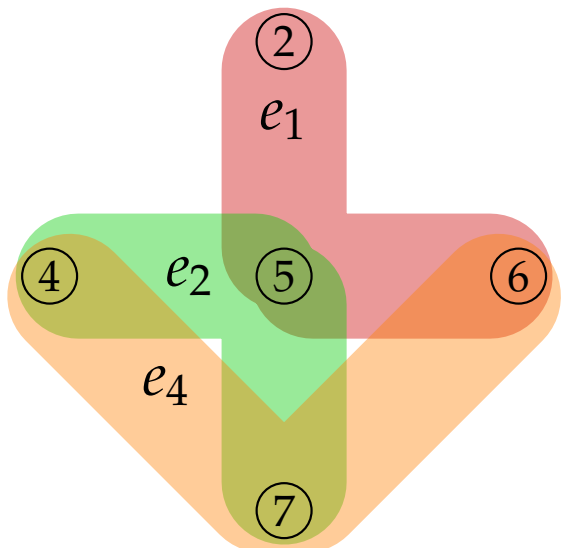

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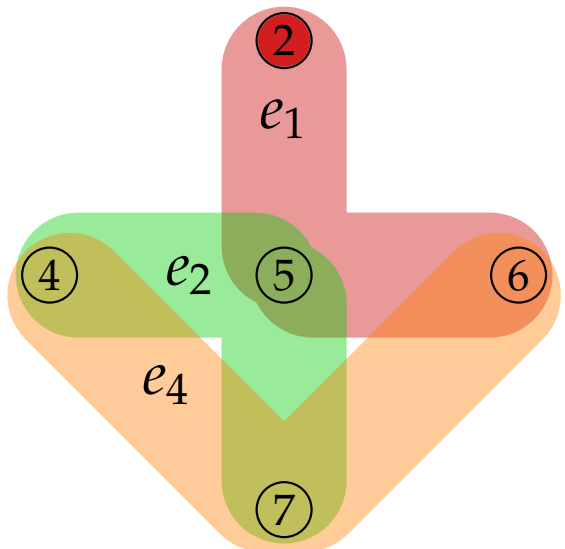

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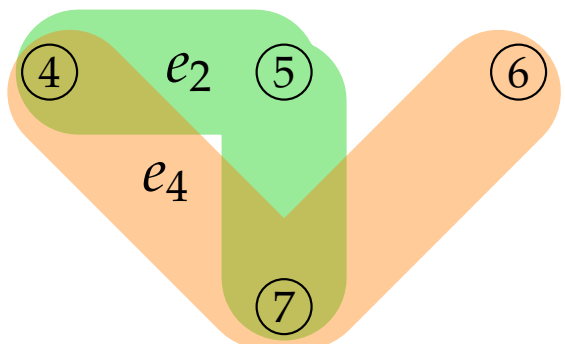

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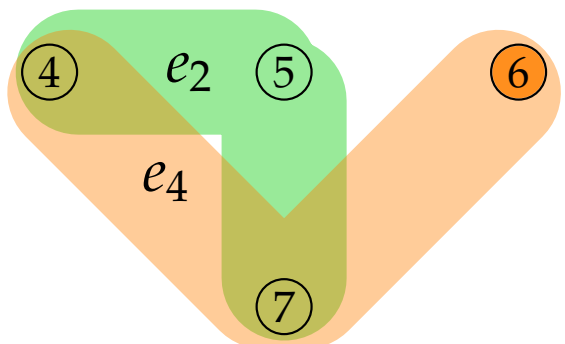

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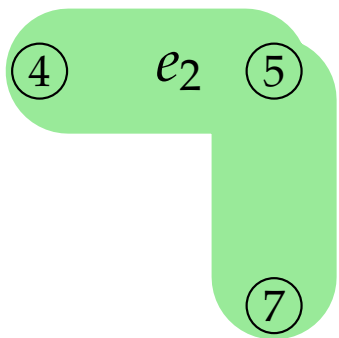

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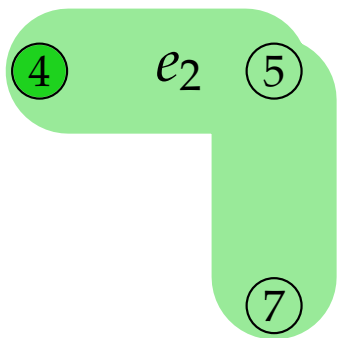

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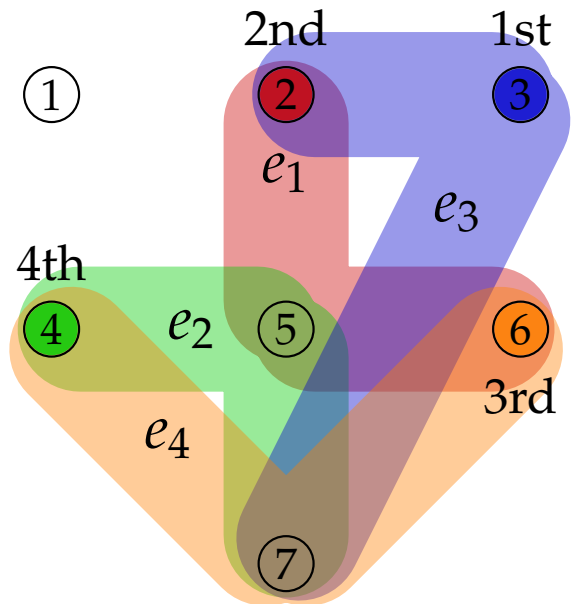

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APPEARANCE OF THE 2-CORE (1)

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- n nodes
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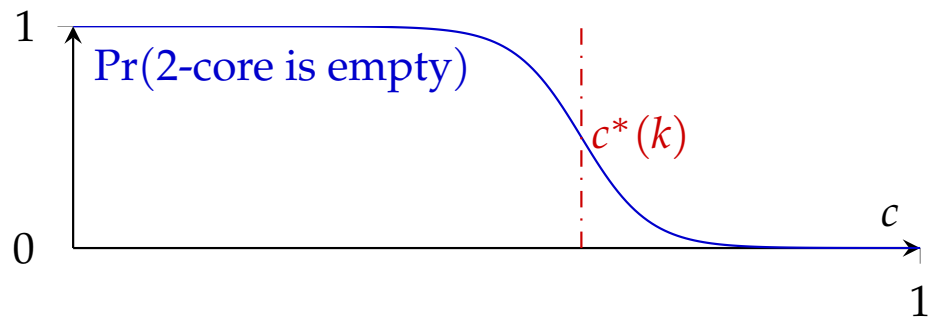
Well known, claim [Majewski et al., 1996], proof [Molloy, 2004]:

- Let $k \geq 3$. For increasing $c = m/n$ there is a *sharp phase transition* from empty to non-empty 2-core.



APPEARANCE OF THE 2-CORE (1)

Model — k-uniform hypergraph:

- n nodes
- m *random* edges of size k from $\binom{\{1,2,\ldots,n\}}{k}$

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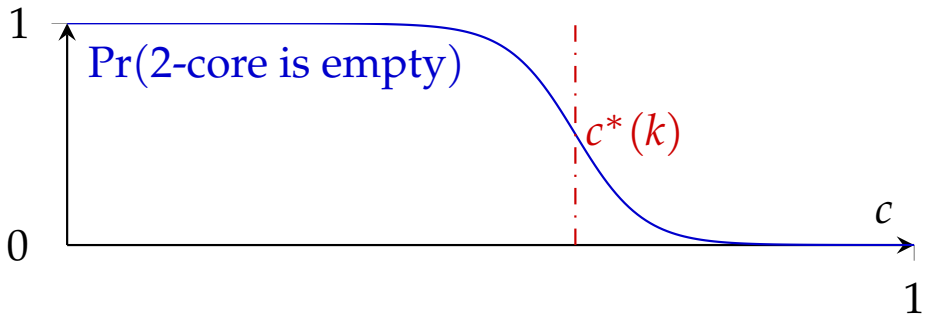
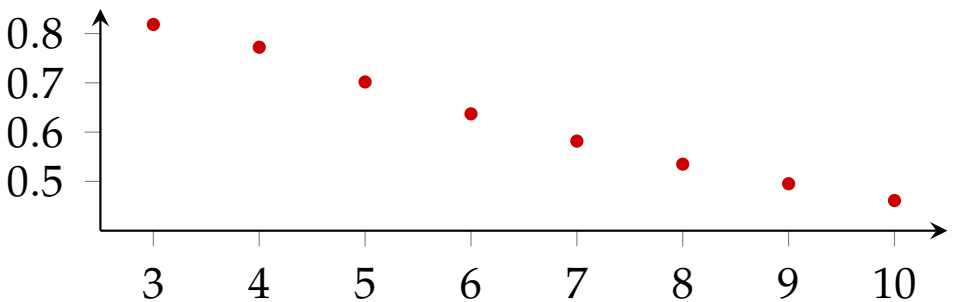


- Thresholds $c^*(k)$, maximum is $c^*(3) \approx 0.818$



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Can we overcome the limit of $c^*(3)$?

Yes, via *mixing* uniform hypergraphs with *different edge sizes*!

Outline

Next . . .

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Application: Perfect Hash Function

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Proof: along the lines of [Molloy, 2004] using ideas from [Dietzfelbinger et al., 2010], see full version

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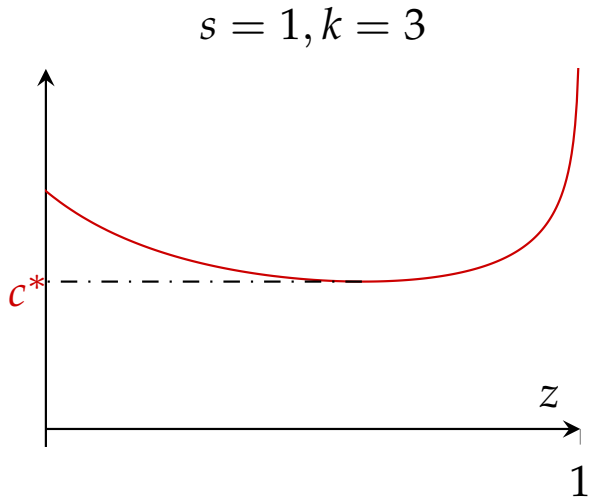

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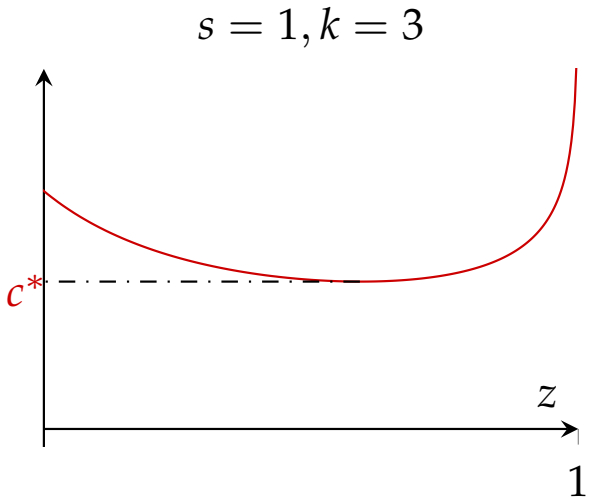

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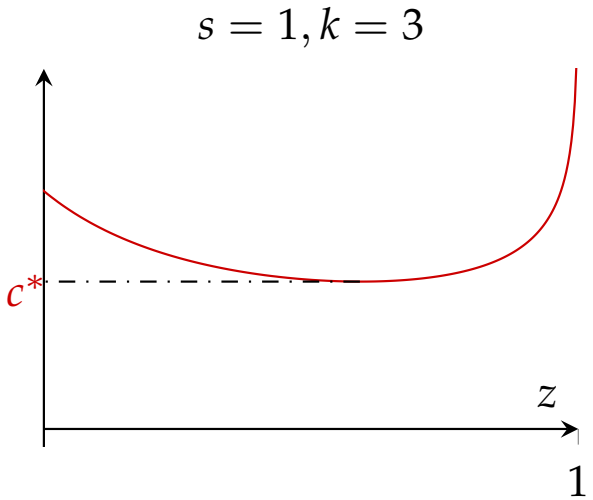

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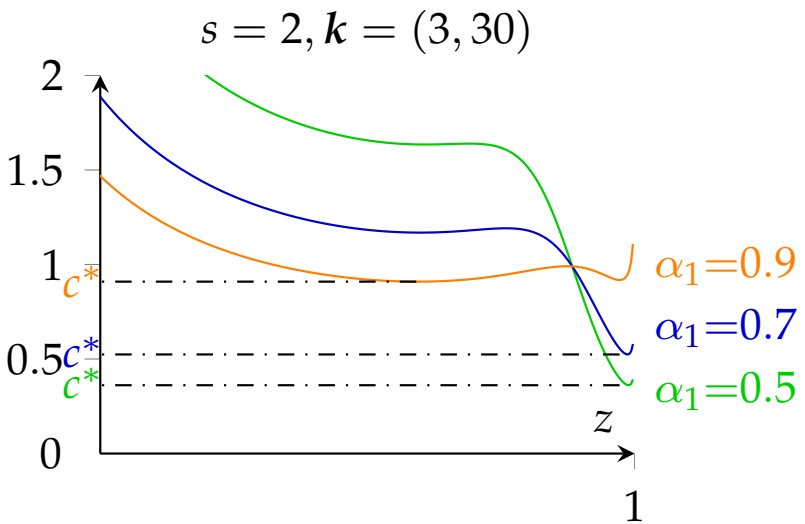

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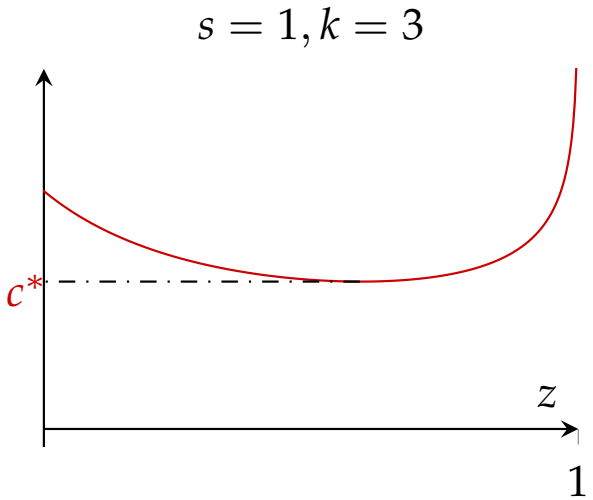

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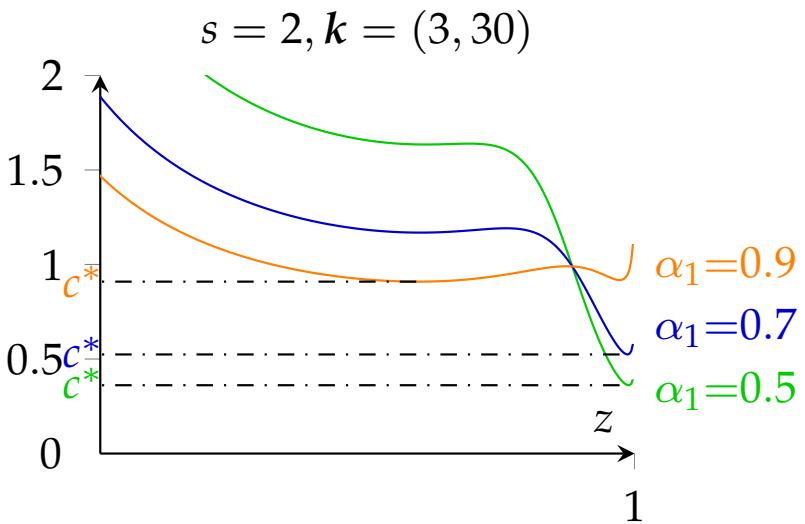

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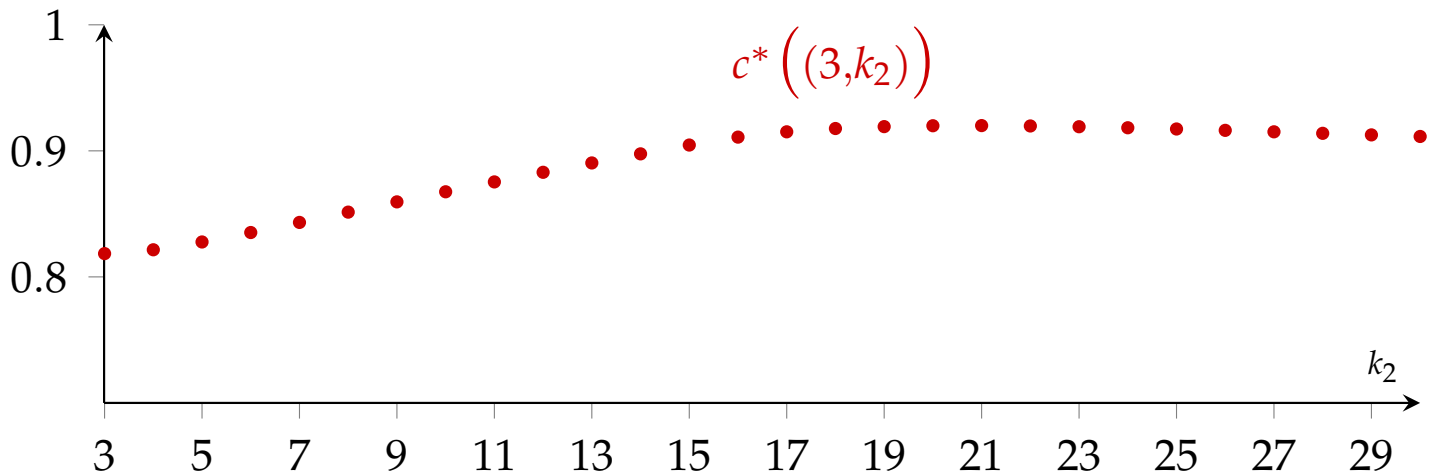

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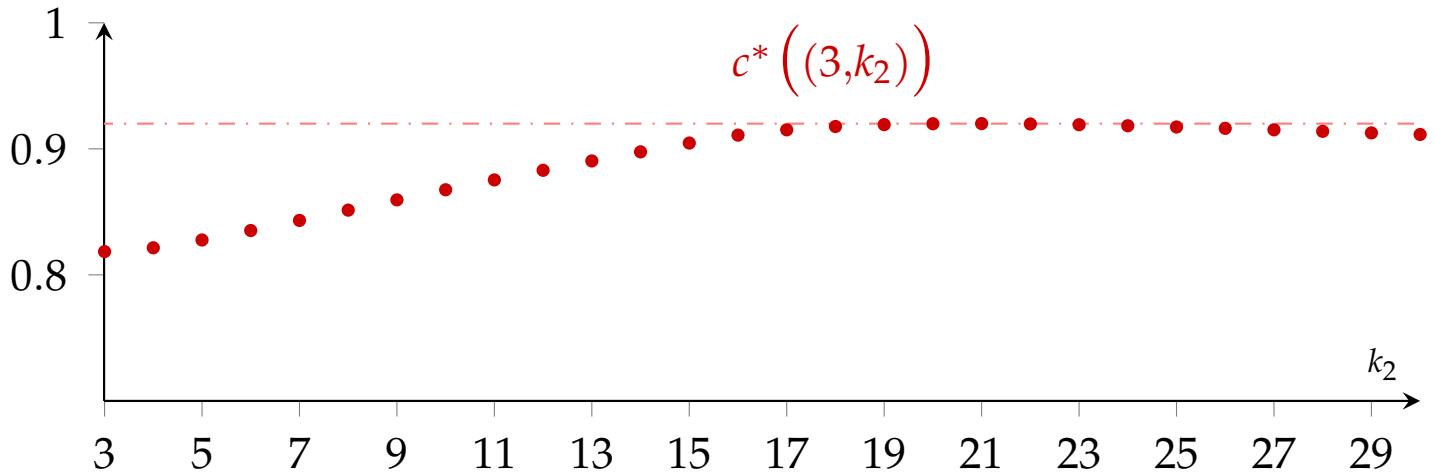

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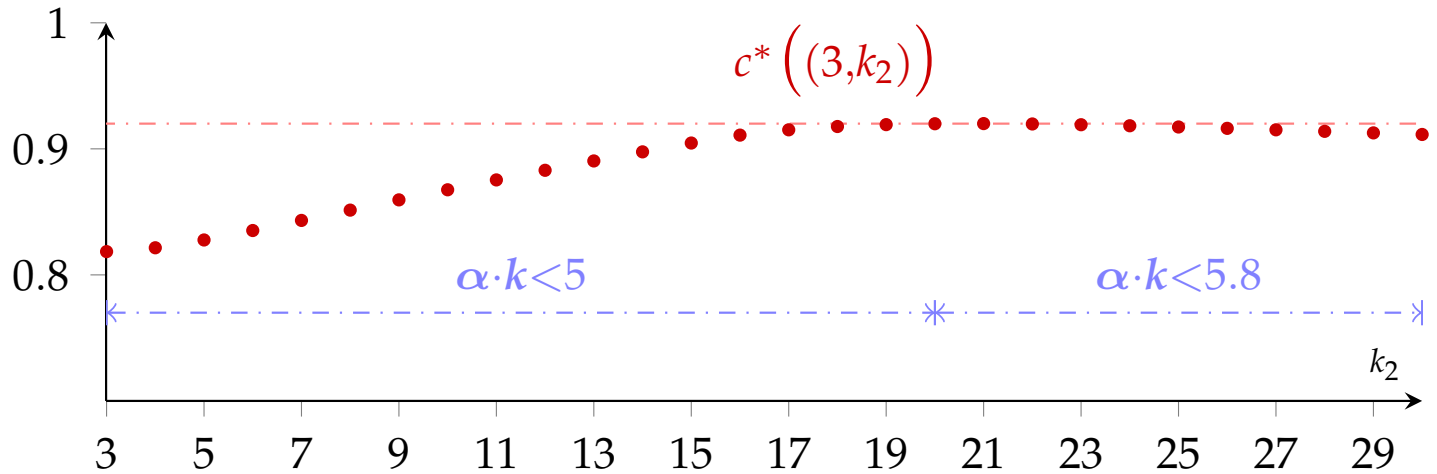

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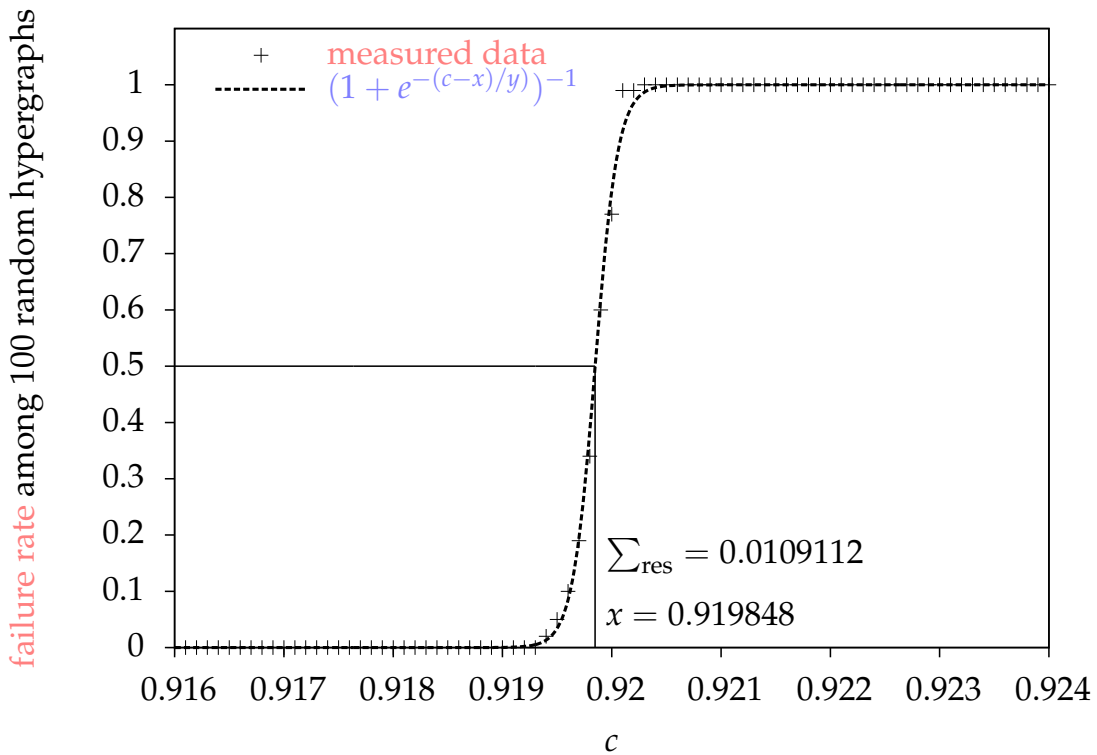

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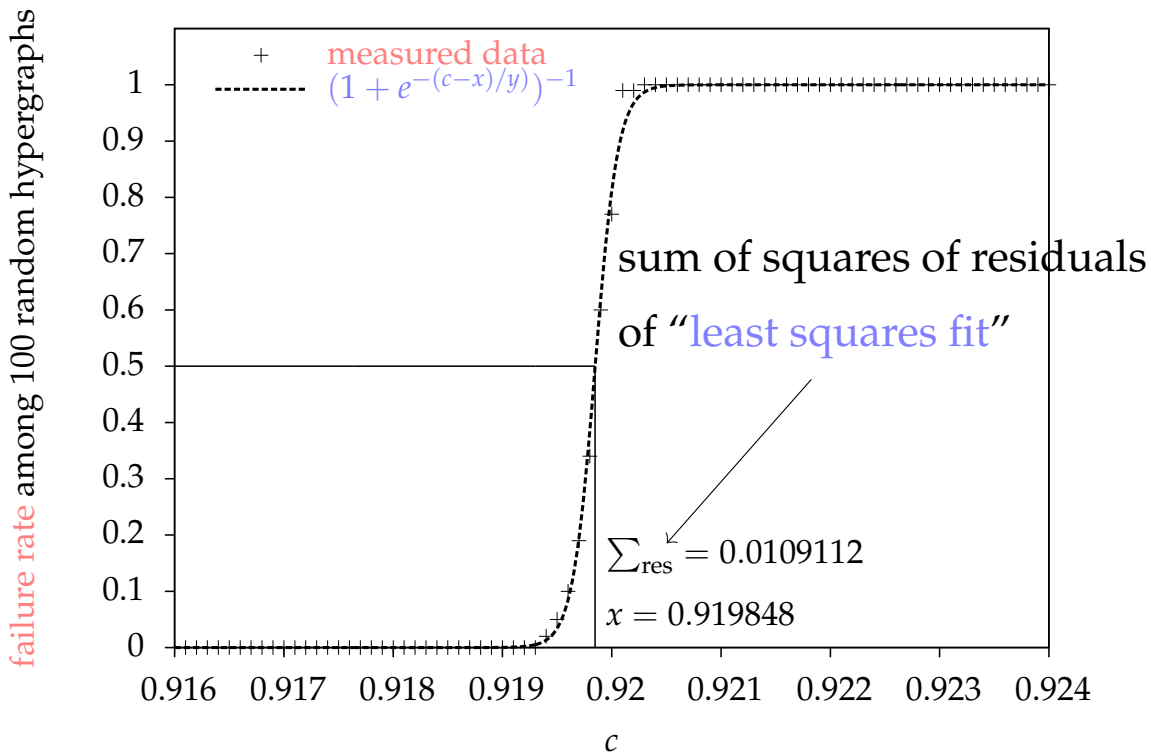

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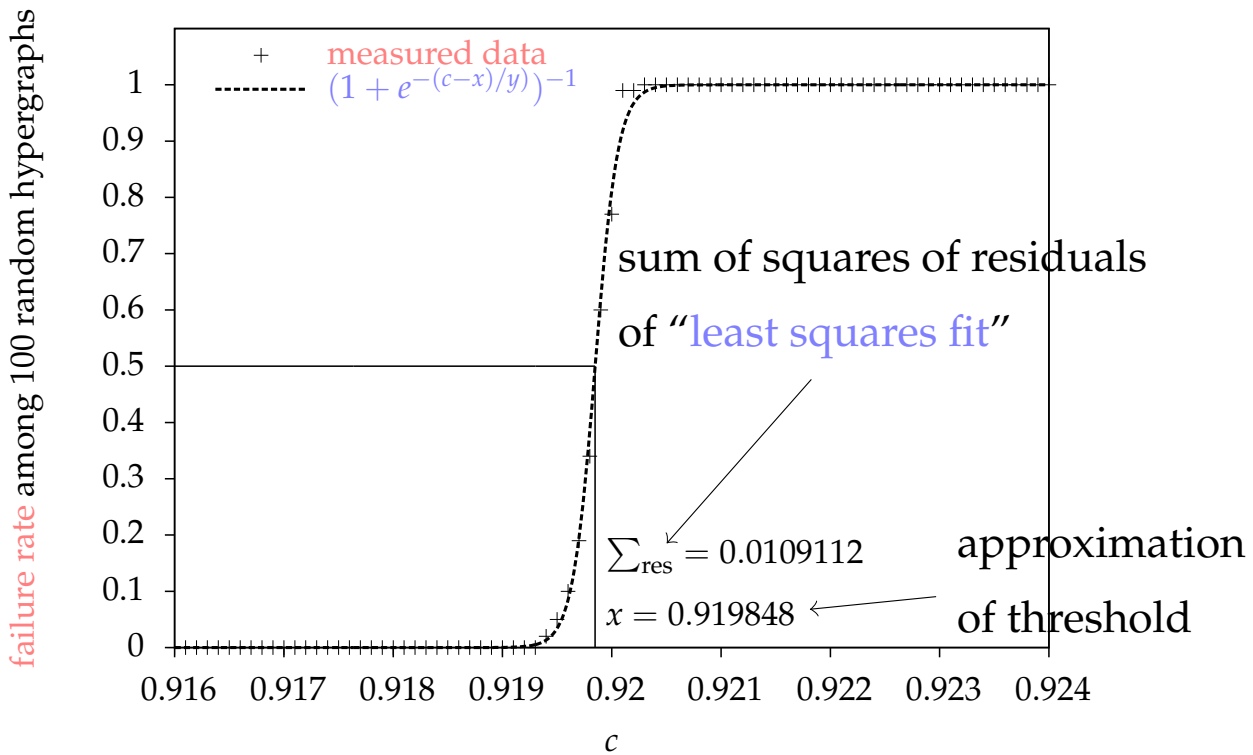

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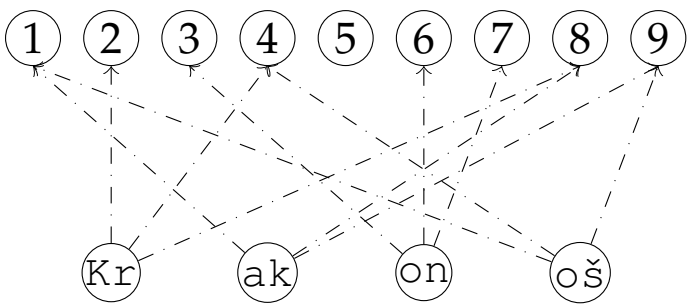

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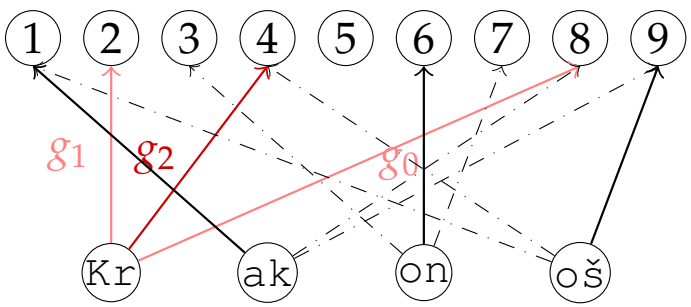

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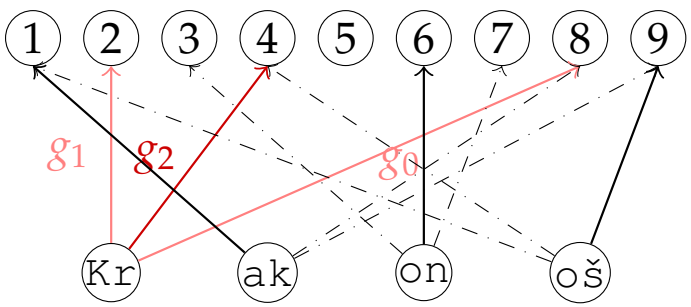

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HIGH LEVEL DESCRIPTION

Problem:

- given set of m keys $S = \{x_1, x_2, \ldots, x_m\}$ and parameter r, usually S is subset of a larger universe U
- build data structure that represents an arbitrary function $f : U \to \{1, 2, \ldots, r\}$, *injective* on S

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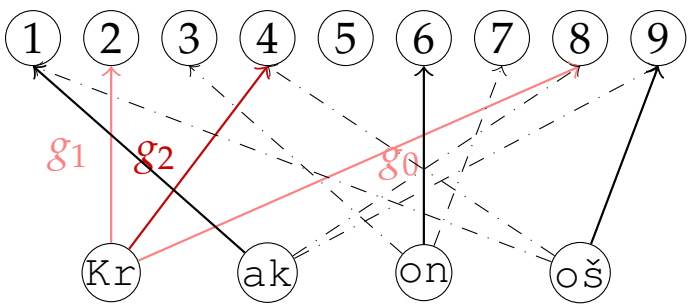

- retrieval: build retrieval data structure for the key-index pairs (x_i, v_i), $i = 1, 2, \ldots, m$, as seen before
- define $f(x) := g_v(x)$, with $v = \text{LOOKUP}(x)$

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Matching (known):

- for $d = 3$ and $r = 1.1 \cdot m$ matching exists whp, e.g.
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Next . . .

Related Work

Results on 2-Cores of Mixed Hypergraphs

Application: Perfect Hash Function

Summary

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- Prove or disprove:

  Conjecture ([Panagiotou, 2012])

  For each $\varepsilon > 0$ there exists $\boldsymbol{k}, \boldsymbol{\alpha}$ such that $c^*(\boldsymbol{k}, \boldsymbol{\alpha}) \geq 1 - \varepsilon$.

Thank you!

Definition of Key Function T(z)

The original "key function" is defined as

$$t(\lambda; \boldsymbol{k}, \boldsymbol{\alpha}) = \frac{\lambda}{\sum_{i=1}^{s} \alpha_i \cdot k_i \cdot \left(\Pr\left(\mathrm{Po}\left[\lambda\right] \geq 1 \right) \right)^{k_i - 1}} ,$$

for $\lambda \in (0, +\infty)$.

We transform $t(\lambda; \boldsymbol{k}, \boldsymbol{\alpha})$ in a more manageable function using a monotonic and bijective domain mapping via $z = 1 - e^{-\lambda}$ and $\lambda = -\ln(1 - z)$.

Hence the transformed "key function" is

$$T(z; \boldsymbol{k}, \boldsymbol{\alpha}) = \frac{-\ln(1 - z)}{\sum_{i=1}^{s} \alpha_i \cdot k_i \cdot z^{k_i - 1}} ,$$

for $z \in (0, 1)$.

Phase 1

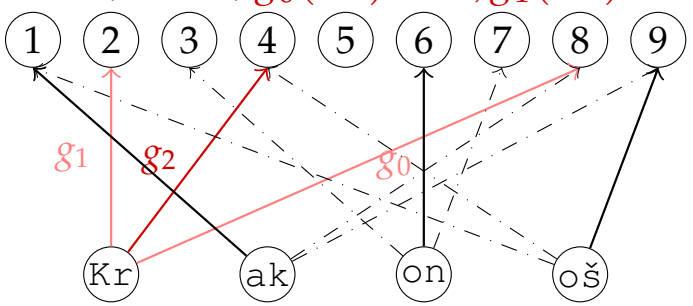
Left-perfect matching:

- given m keys $x_1, x_2, \ldots, x_m$ and number r

  $m = 4, r = 9, x_1 = \texttt{Kr}, x_2 = \texttt{ak}, x_3 = \texttt{on}, x_4 = \texttt{oš}$

- build bipartite graph $\mathcal{B}$
  - node sets $\{x_1, x_2, \ldots, x_m\}$ and $\{1, 2, \ldots, r\}$
  - edges given via d random hash functions $g_0, g_1, \ldots, g_{d-1}$

    $d = 3, r = 9, g_0(\texttt{Kr}) = 2, g_1(\texttt{Kr}) = 4, g_2(\texttt{Kr}) = 8$



- determine matching in $\mathcal{B}$, for each matching edge $\{x_i, g_{v_i}(x_i)\}$ temporarily store key-index pair (x_i, v_i)

  $(\texttt{Kr}, 2), (\texttt{ak}, 0), (\texttt{on}, 1), (\texttt{oš}, 0)$

PHASE 2

Retrieval:

- given key-index pairs (x_i, v_i), $i = 1, 2, \ldots, m$, and group $(\mathbb{Z}_d, +)$

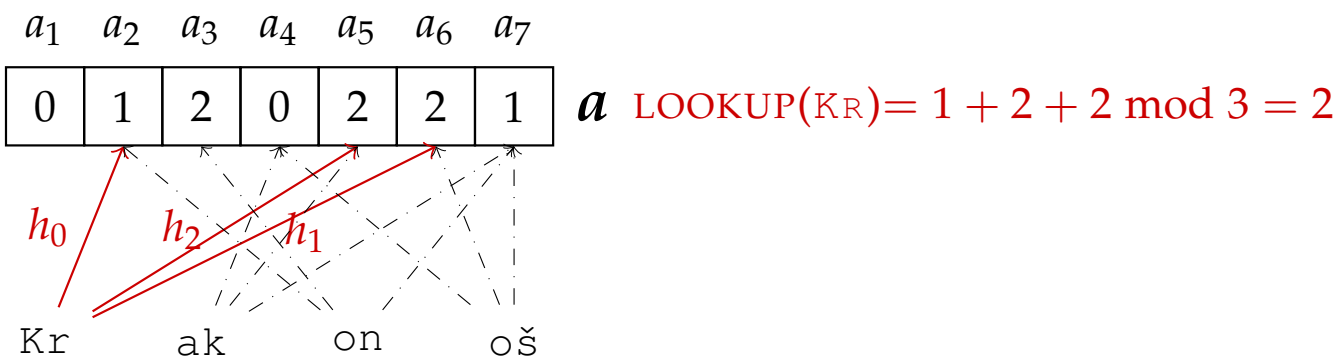
  $(\texttt{Kr}, 2), (\texttt{ak}, 0), (\texttt{on}, 1), (\texttt{oš}, 0), (\mathbb{Z}_3, +)$

- build binary $m \times n$ matrix $\boldsymbol{M}$, where the position of ones in row i are given via the values of random hash functions $h_0, h_1, \ldots, h_{k-1}$ for key x_i

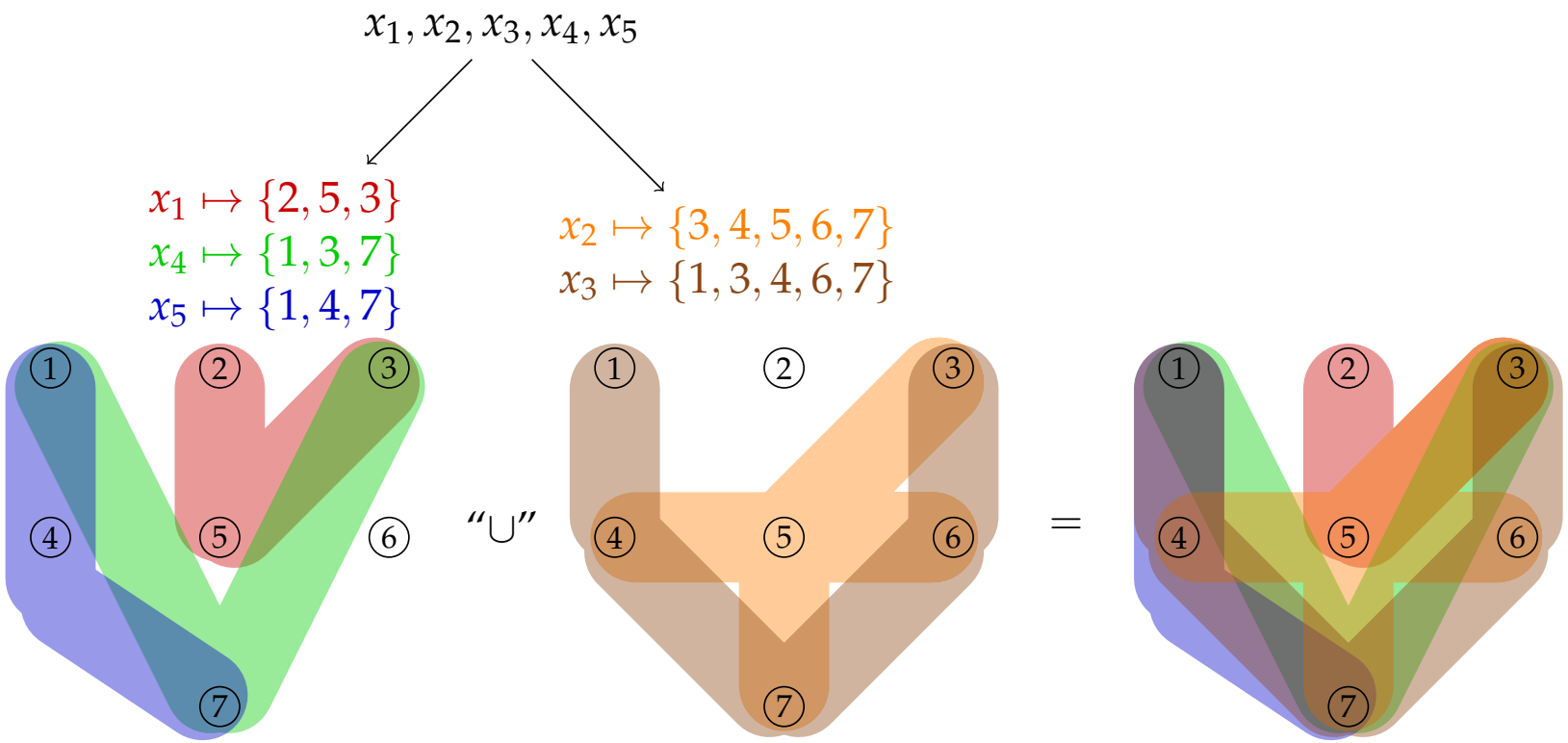
  $k = 3, n = 7, h_0(\texttt{Kr}) = 2, h_1(\texttt{Kr}) = 6, h_2(\texttt{Kr}) = 5 \Rightarrow \texttt{Kr} \mapsto (0\,1\,0\,0\,1\,1\,0)$

- solve $\boldsymbol{M} \cdot \boldsymbol{a} = \boldsymbol{v}$

- $f := g_v(x)$, where
  $v = \text{LOOKUP}(x) := a_{h_0(x)} \oplus a_{h_1(x)} \oplus \cdots \oplus a_{h_{k-1}(x)}$



Mixed Hypergraphs



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