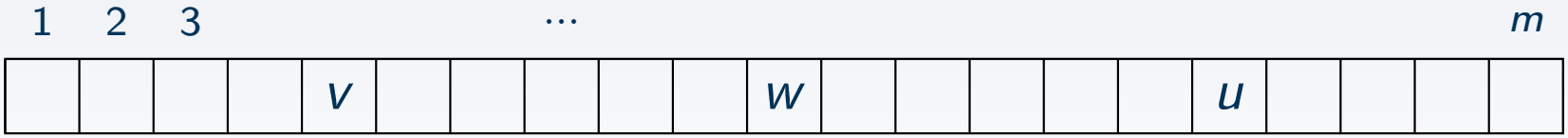
Cuckoo Hashing with Pages

Martin Dietzfelbinger Technische Universität Ilmenau
Michael Mitzenmacher Harvard University
Michael Rink Technische Universität Ilmenau

07.09.11

k-ary Cuckoo Hashing



[Fotakis et al., 2005]:

- n keys, table of size m

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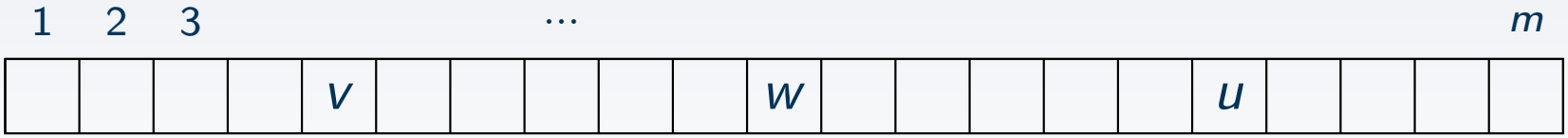

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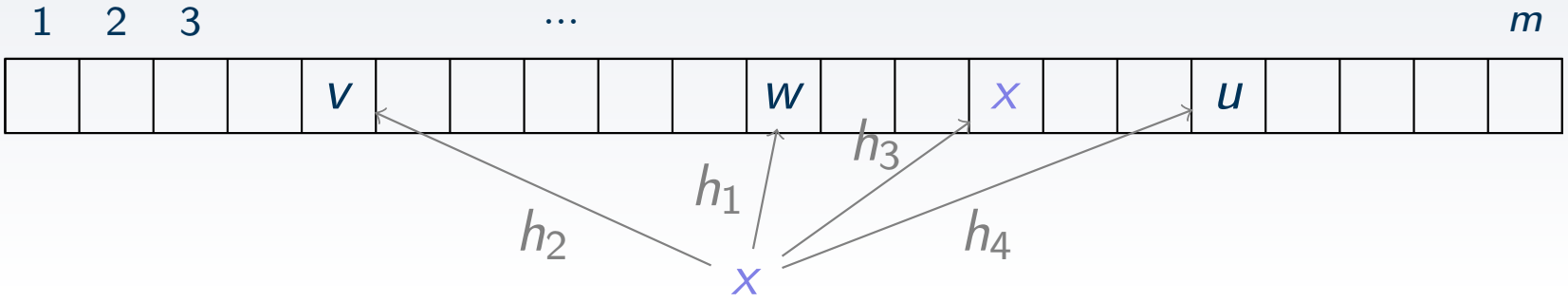

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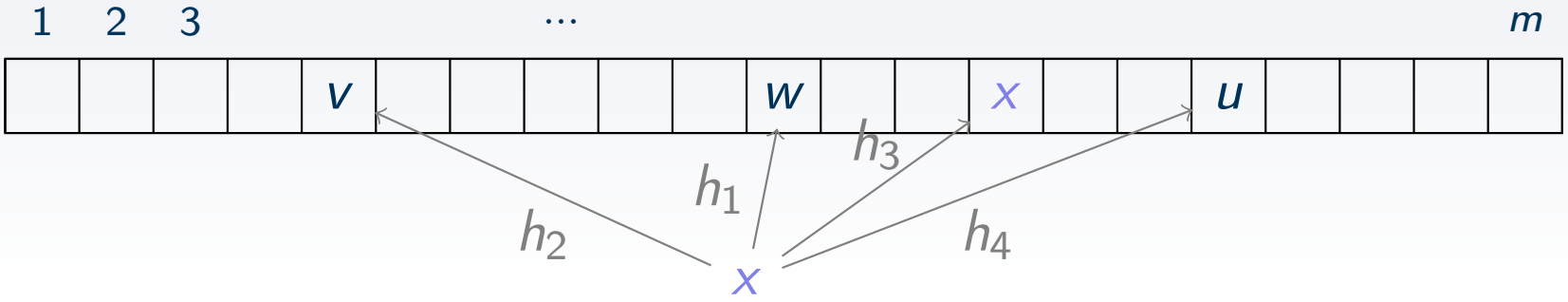

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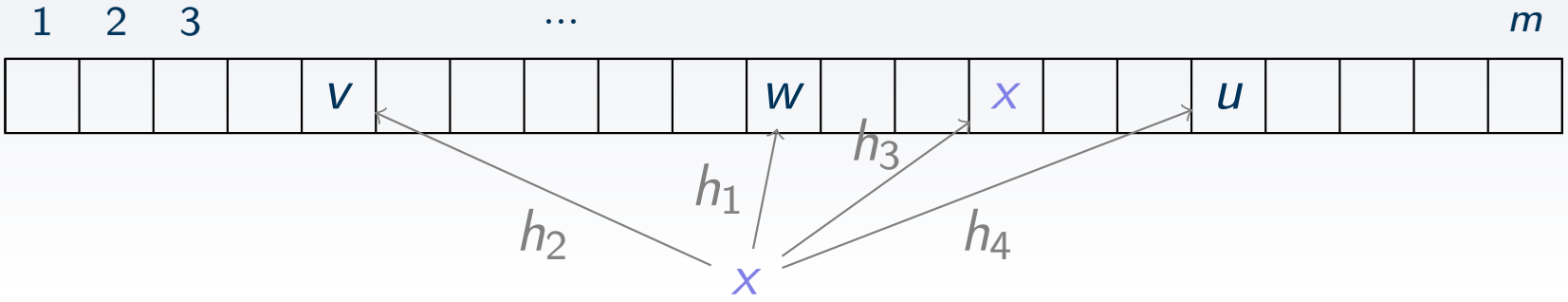

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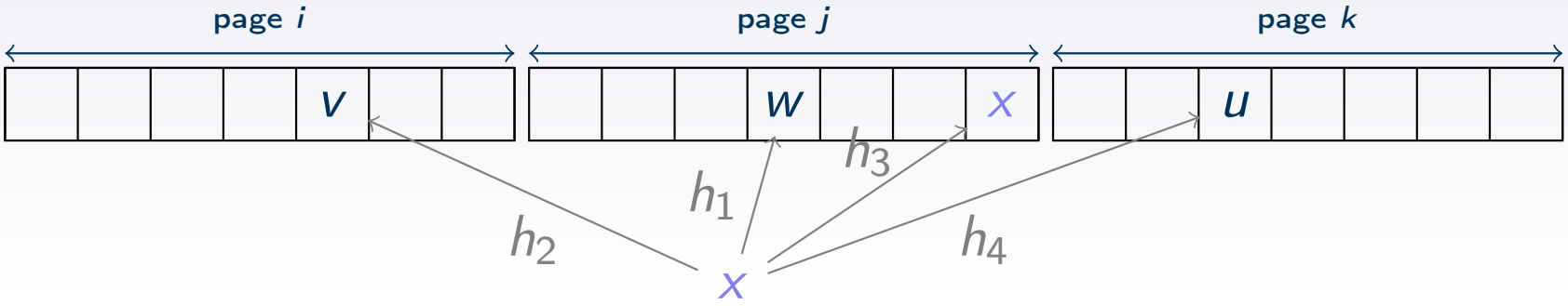
Downside:

k-ary Cuckoo Hashing



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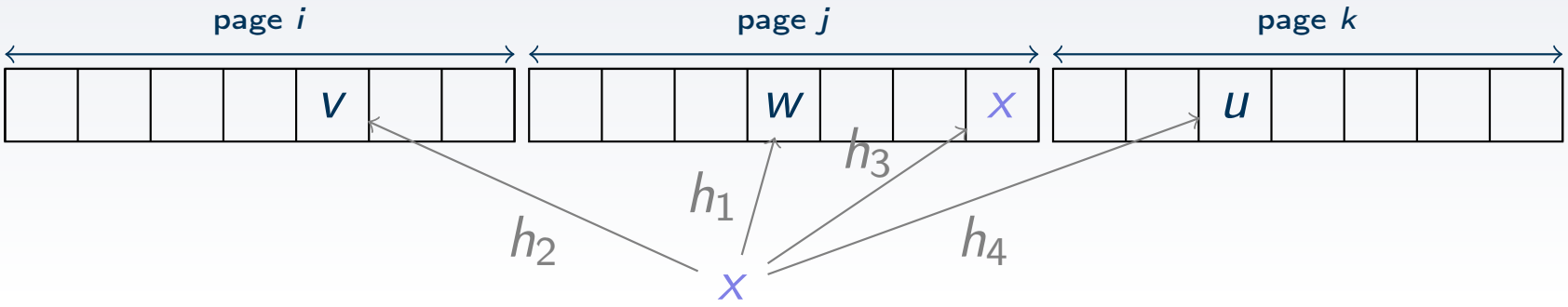

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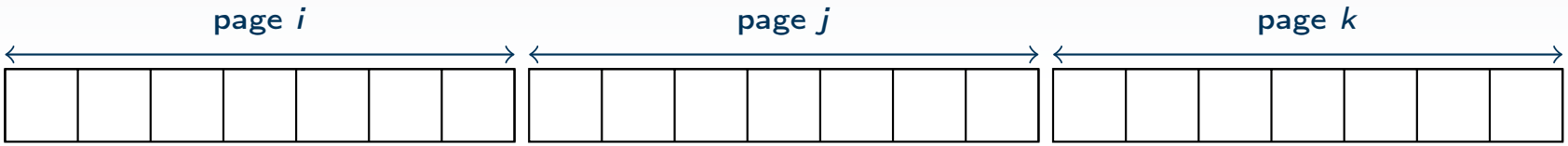
Downside: if memory arranged in pages + costly page requests $\longrightarrow$ **expensive lookups**

Dealing with Pages (1)

Natural approach:



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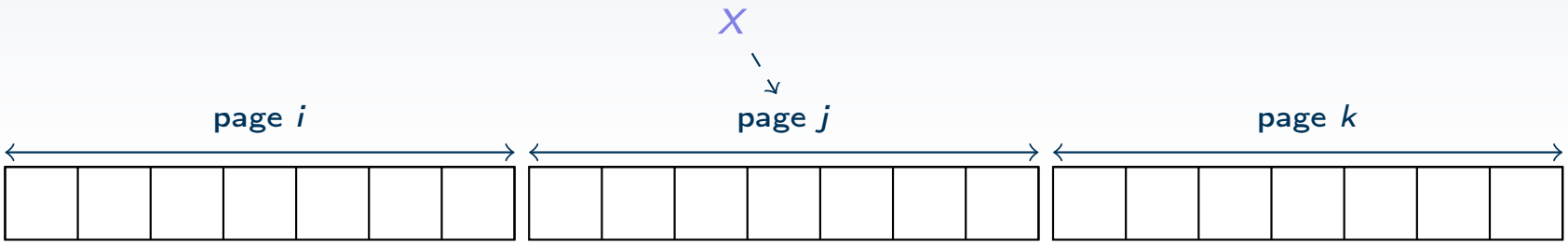

- map key to a page

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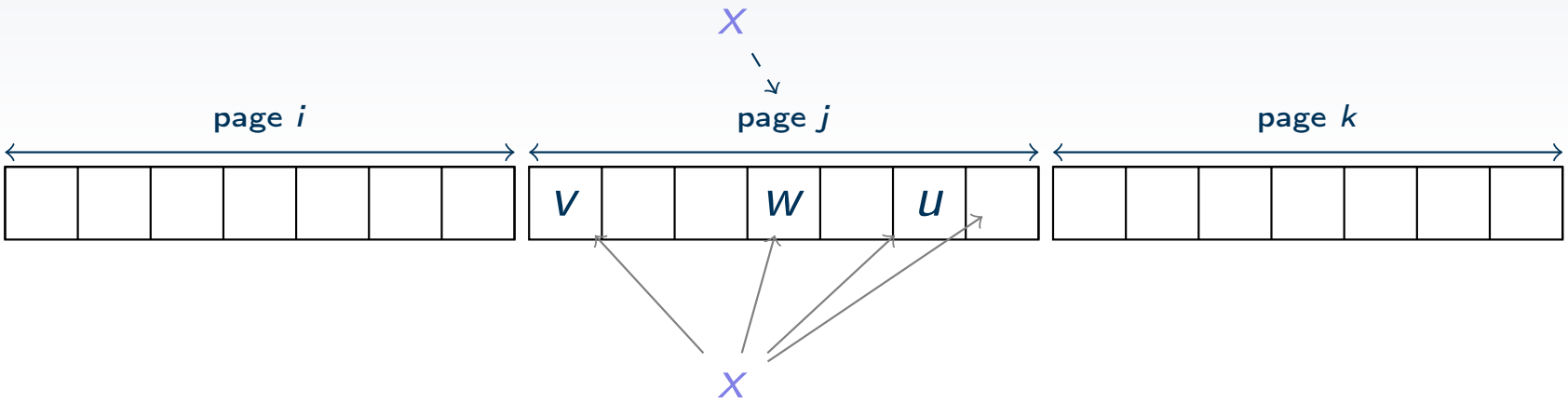

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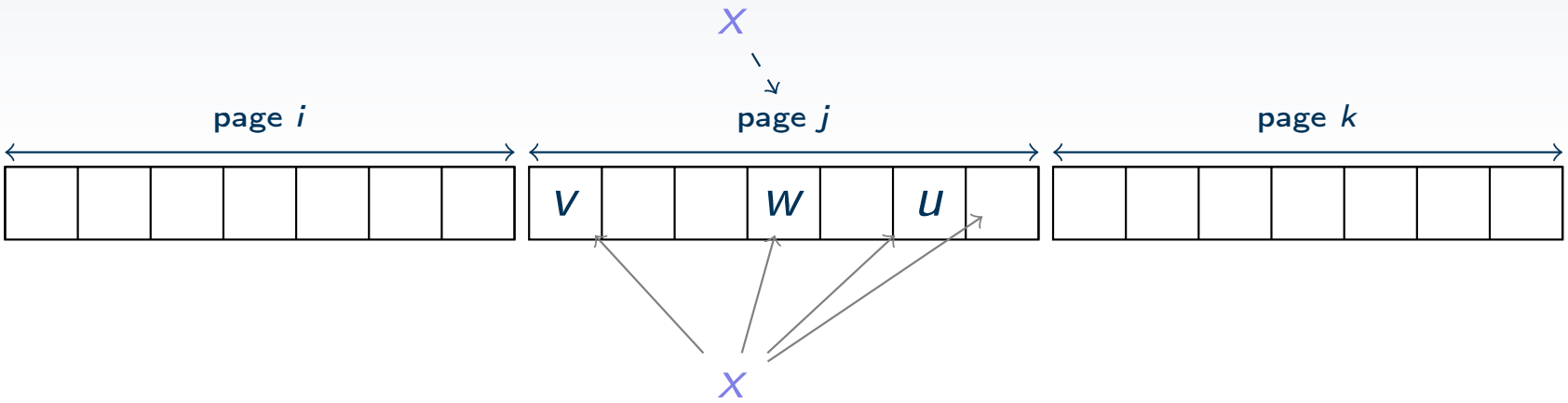

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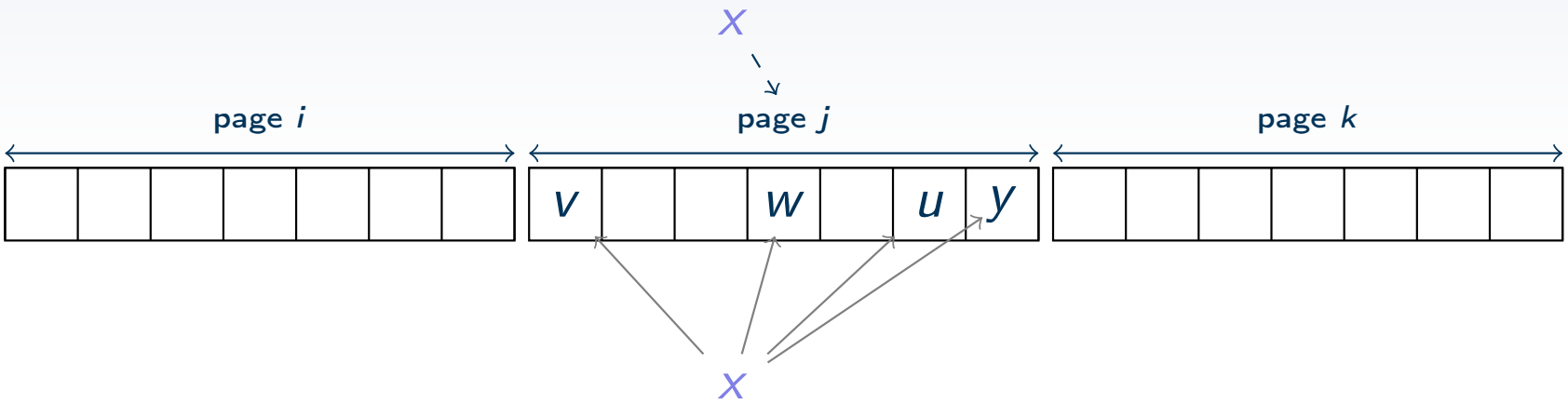
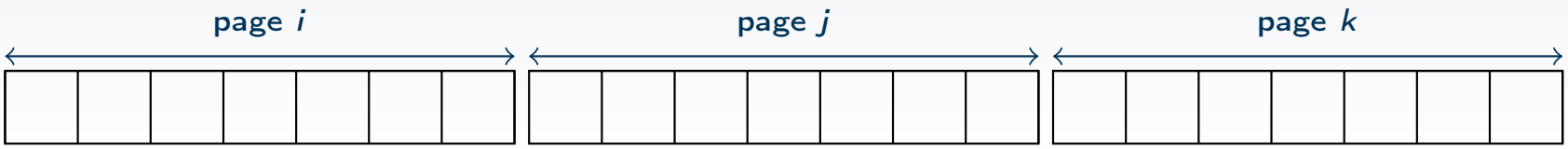


- map key to a page
- separate cuckoo hash (sub-)table for each page
- **Pro:** only 1 page request
- **Con:** most overloaded sub-table limits load of overall table

Dealing with Pages (2)

Our approach:



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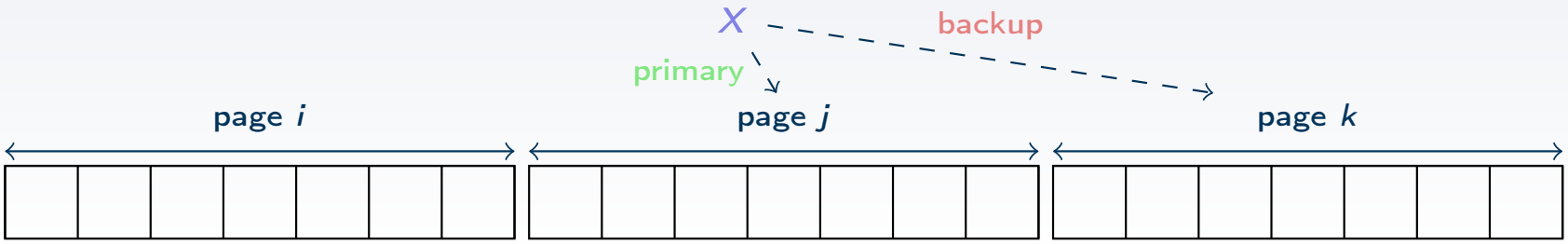

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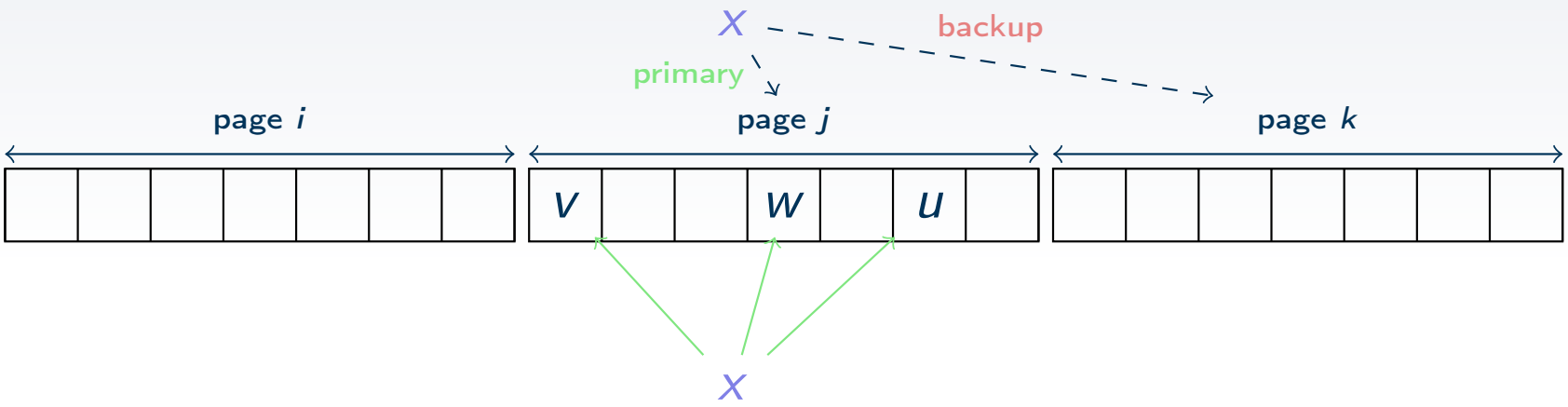

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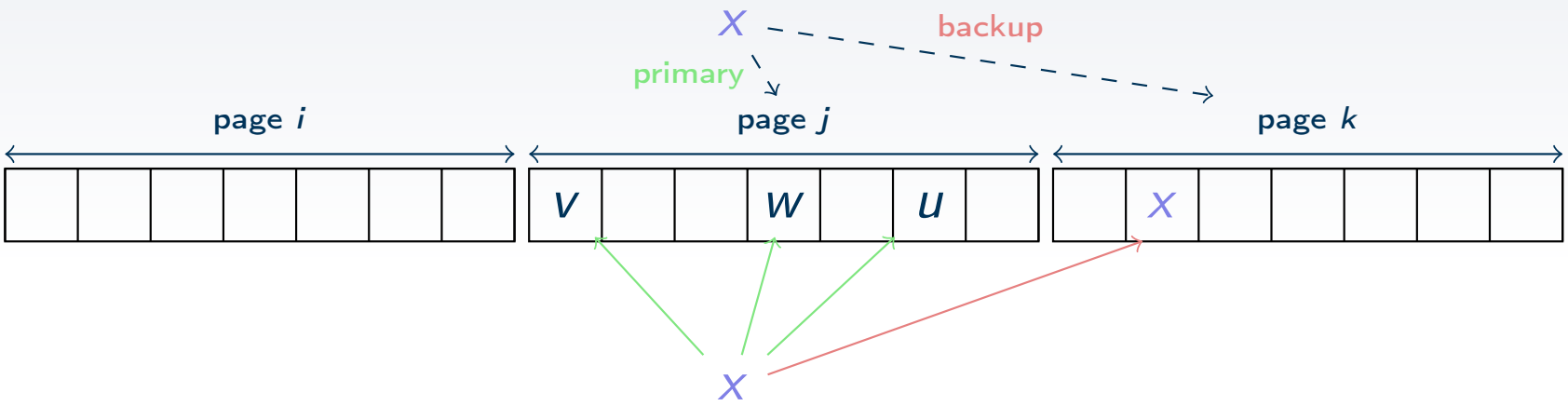

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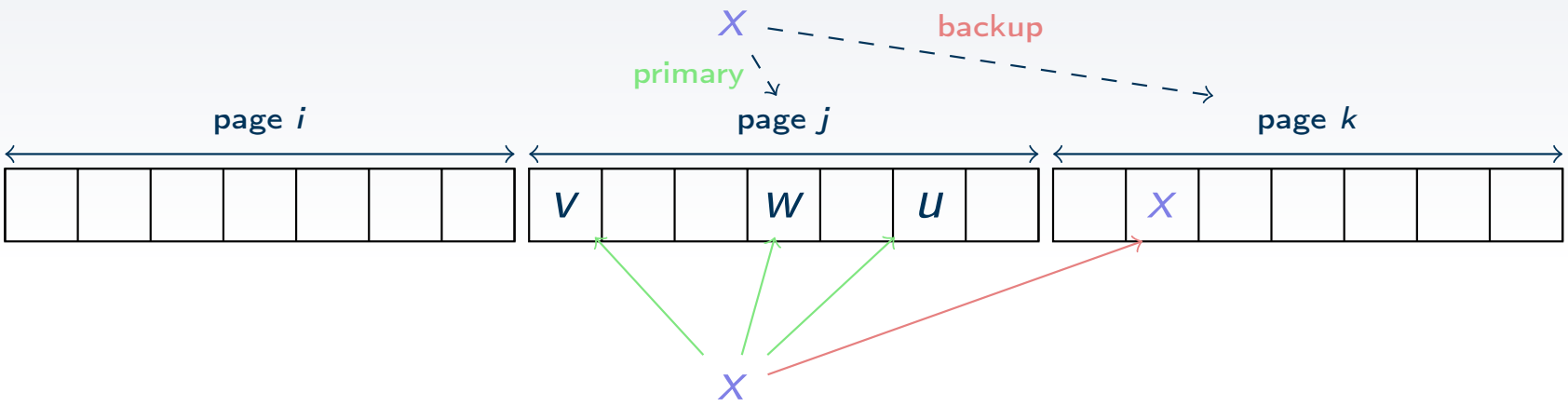

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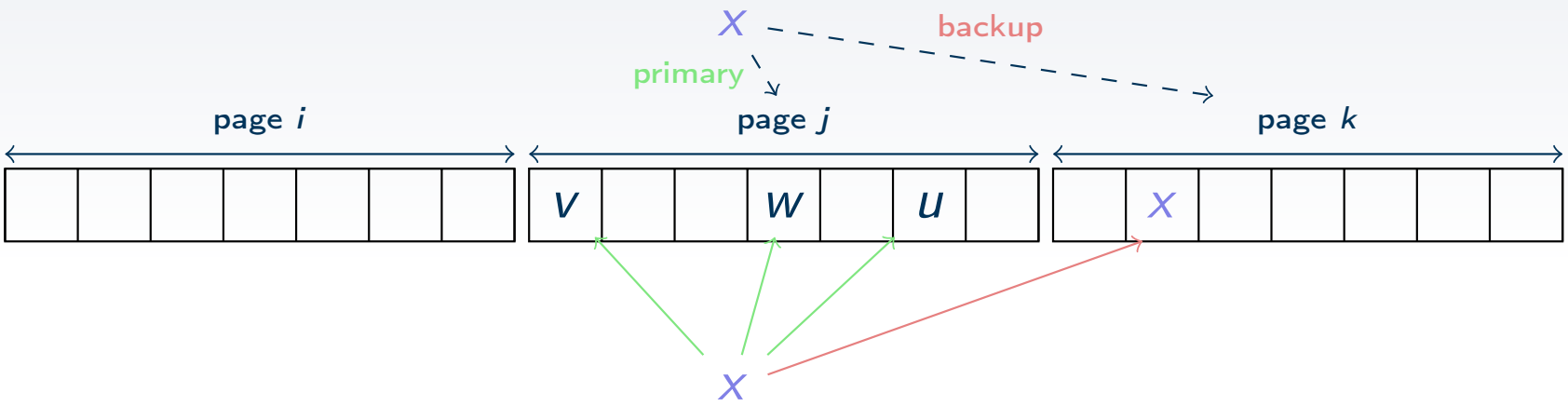

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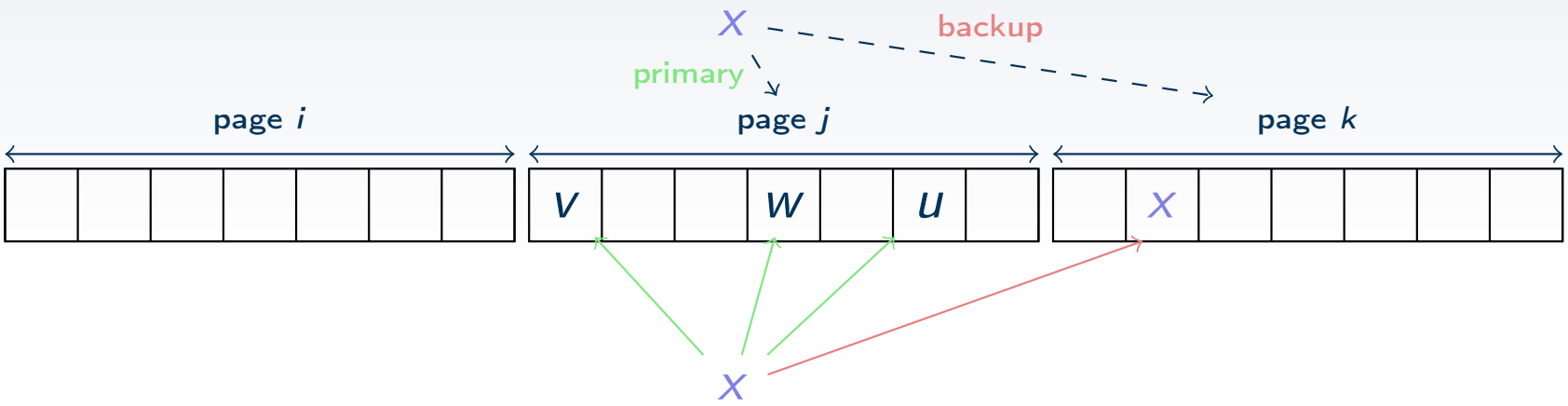

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Work is entirely experimental!

(Some) Related Work

1 page, k choices, page size $s = m^{\delta}$

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[Frieze and Melsted, 2009, arXiv]
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- 1 page request for $1 - \varepsilon$ fraction of keys
- additional perfect hashing scheme for each primary page needed

Experiments

Setup:

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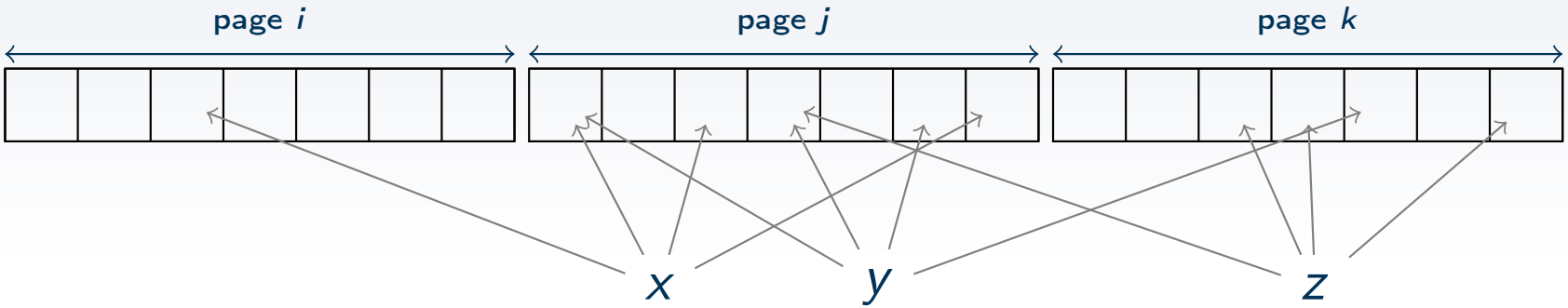
Results:

- averages among 100 attempts

Static Case



Natural Approach:

- assign costs to the edges of the bipartite cuckoo graph

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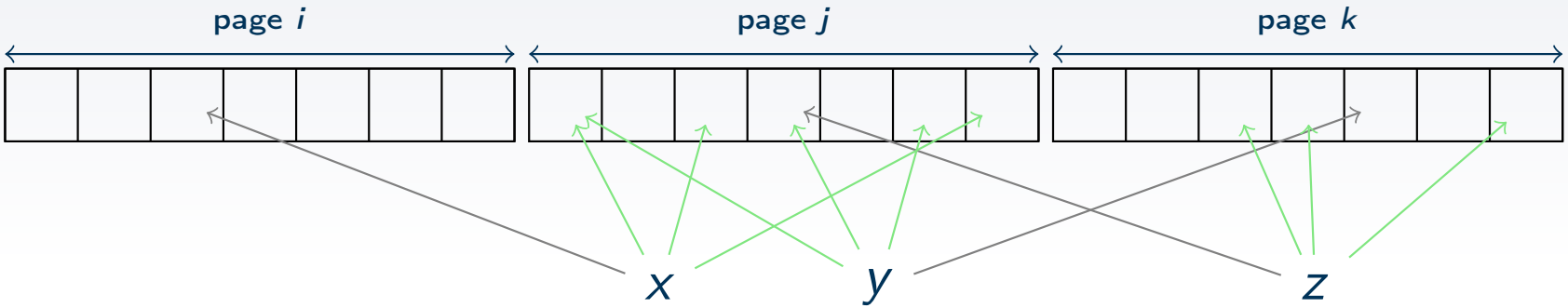

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- primary edges have cost 0

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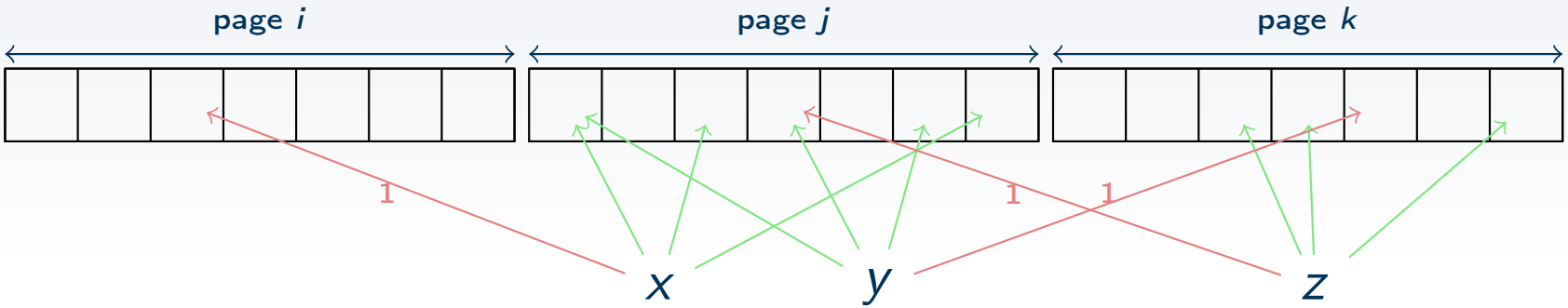

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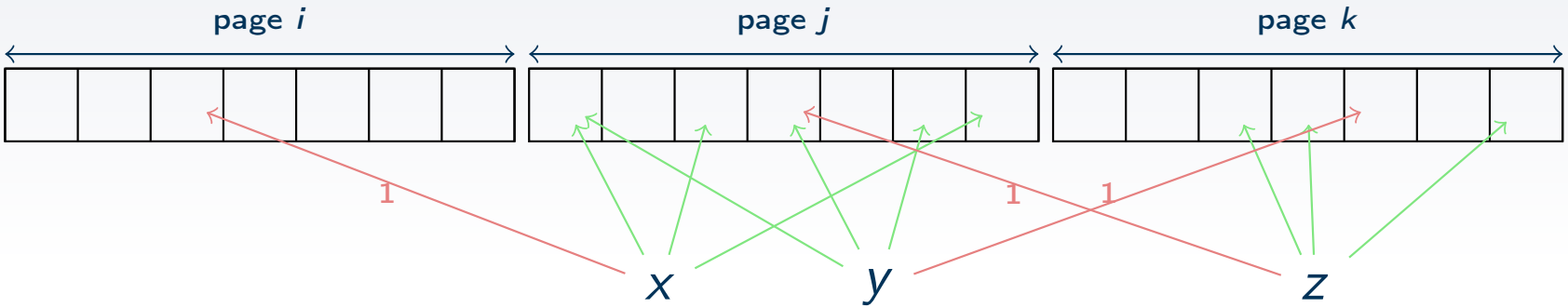

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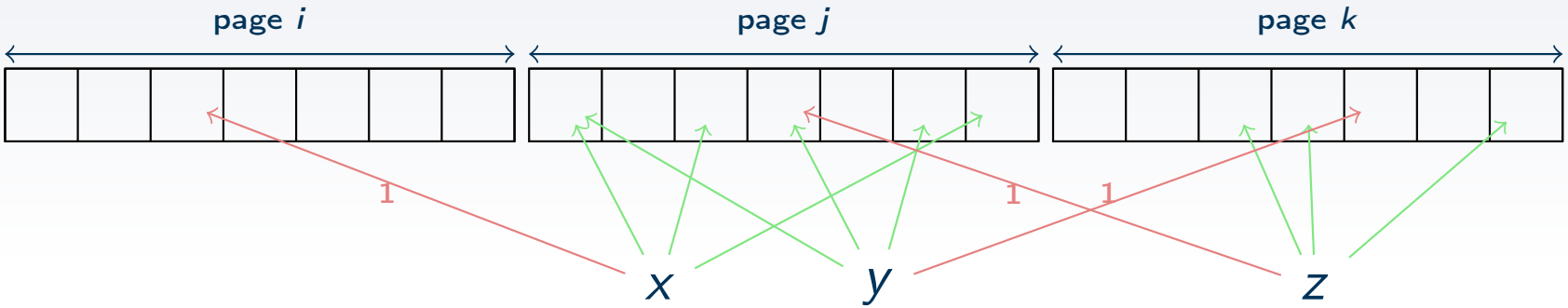

Natural Approach:

- assign costs to the edges of the bipartite cuckoo graph
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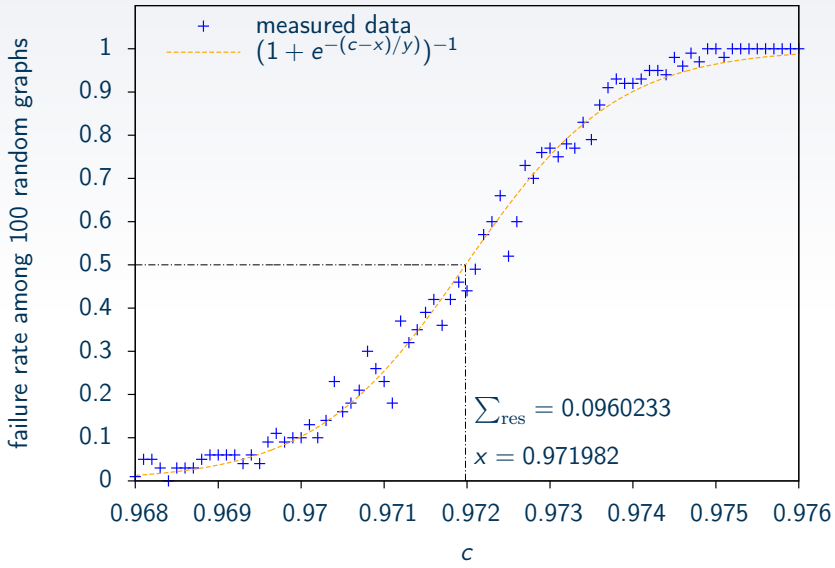
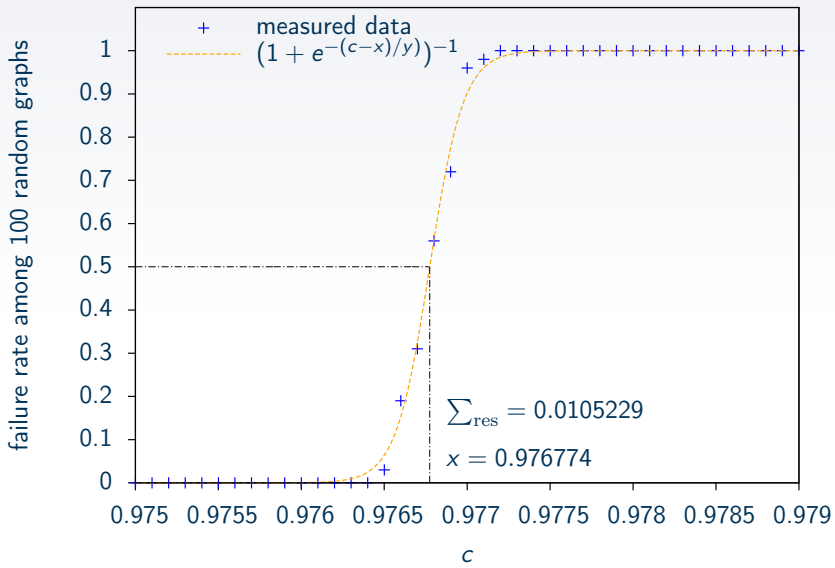
- solve minimum cost matching problem
- many algorithms available — used a successive shortest path algorithm (see full version on arXiv)

Comparison of Maximum Load

4 choices, 1 page (no backup)



3 primary, 1 backup choice



Threshold:

page size	4 choices, 1 page (no backup)	3 primary, 1 backup choice
10^5	0.972	0.977

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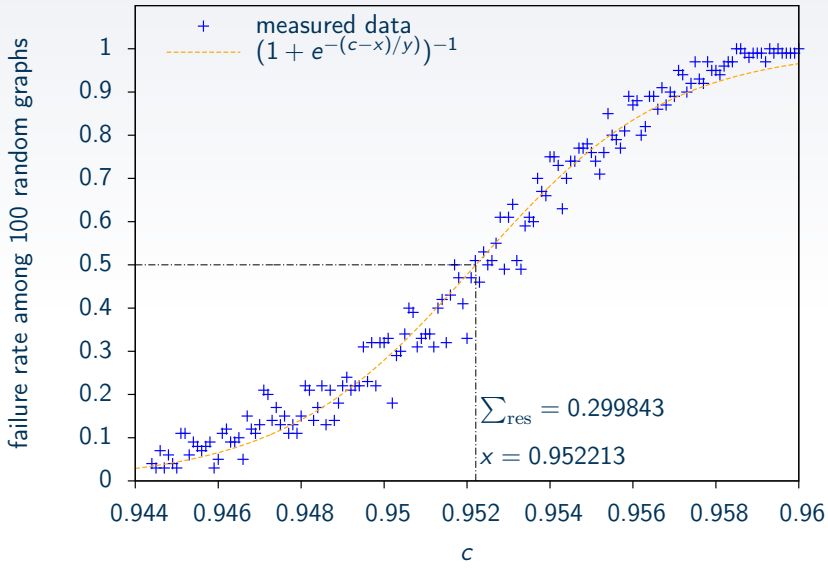

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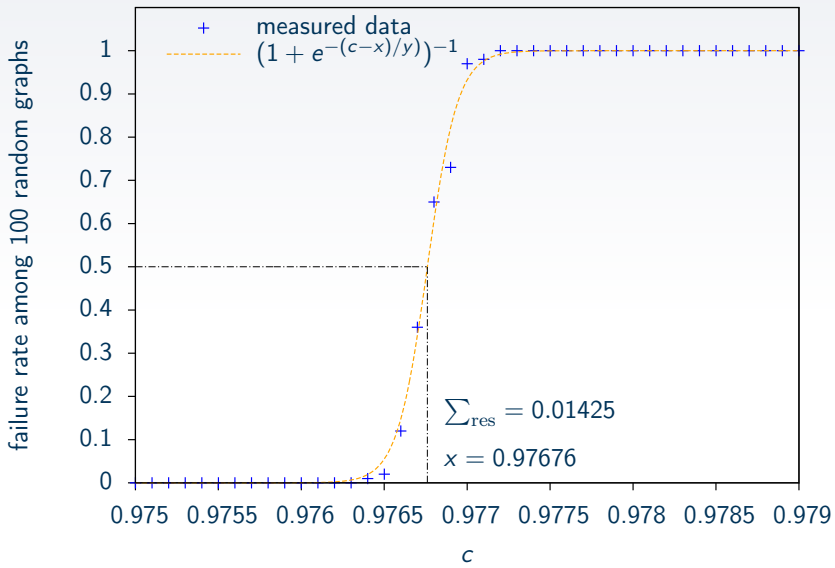

←– interval grows –→

Threshold:

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10^4	0.952	0.977

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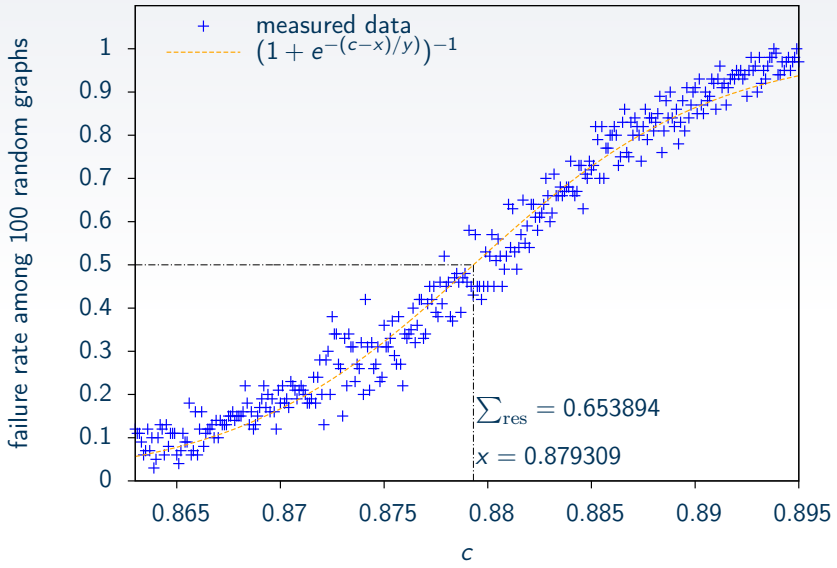

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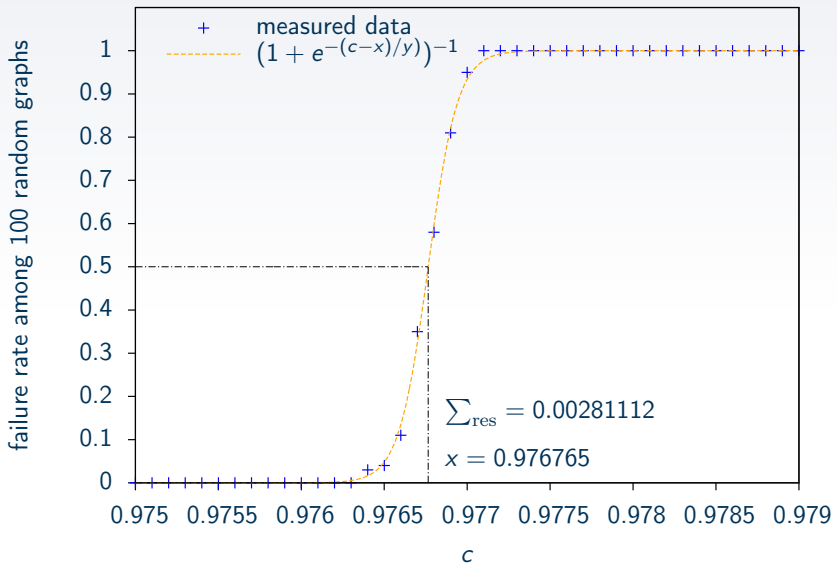

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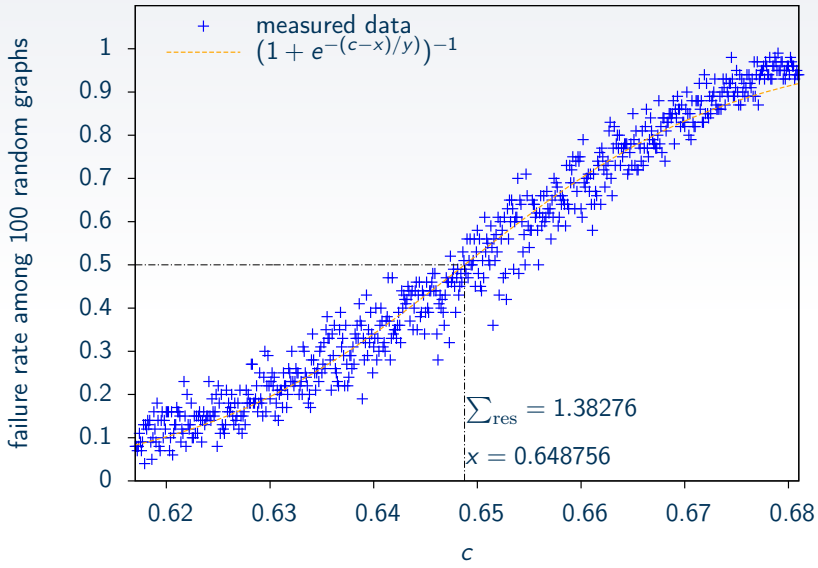

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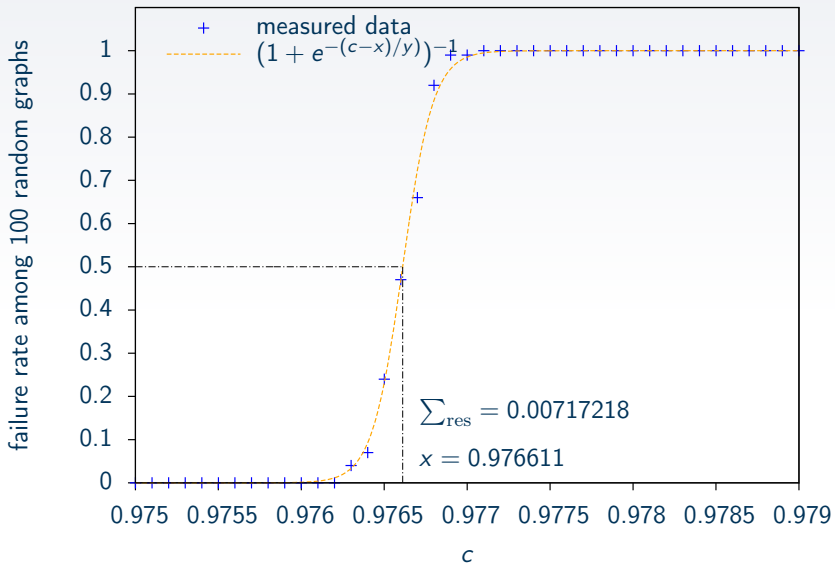

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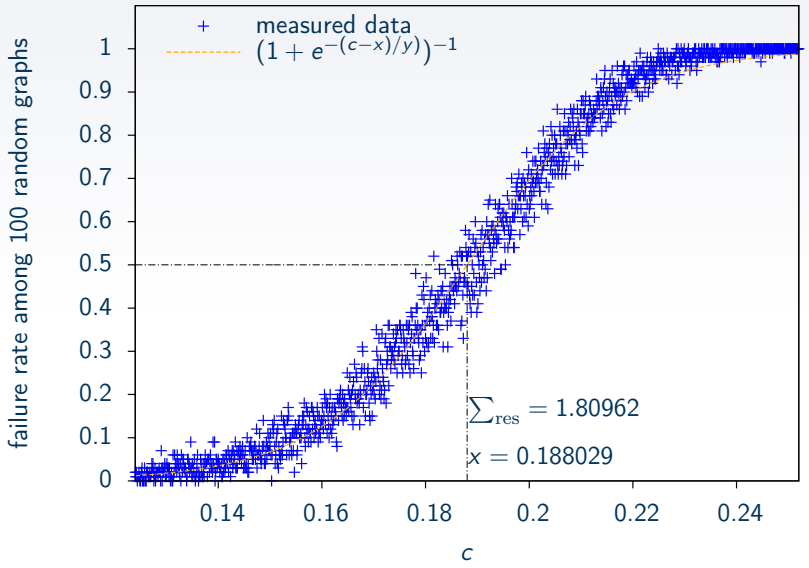

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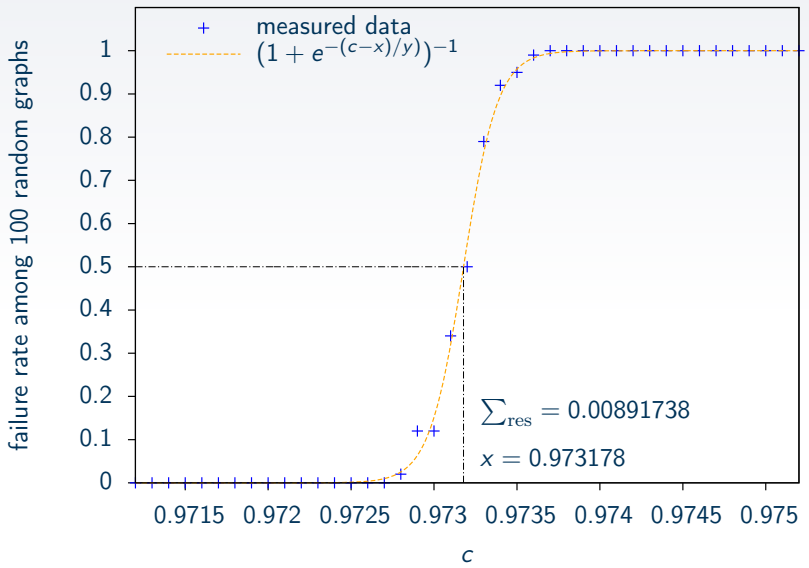

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⟵ interval shift

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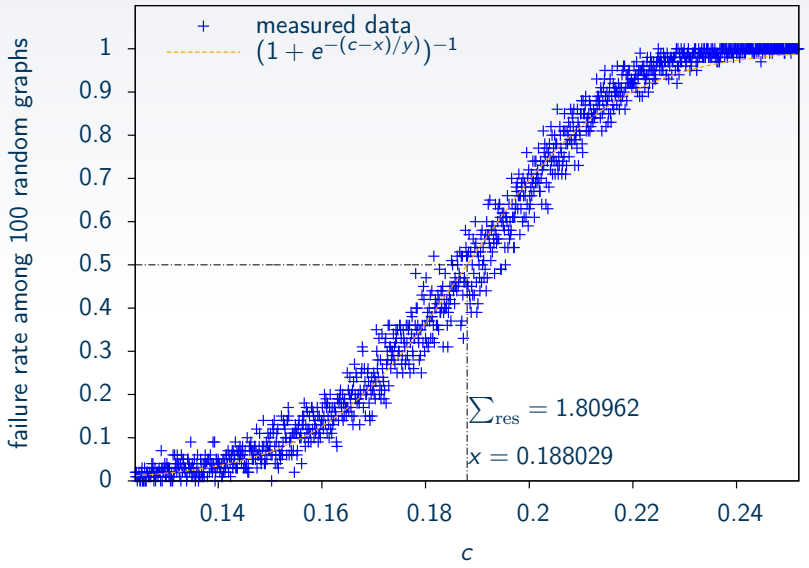
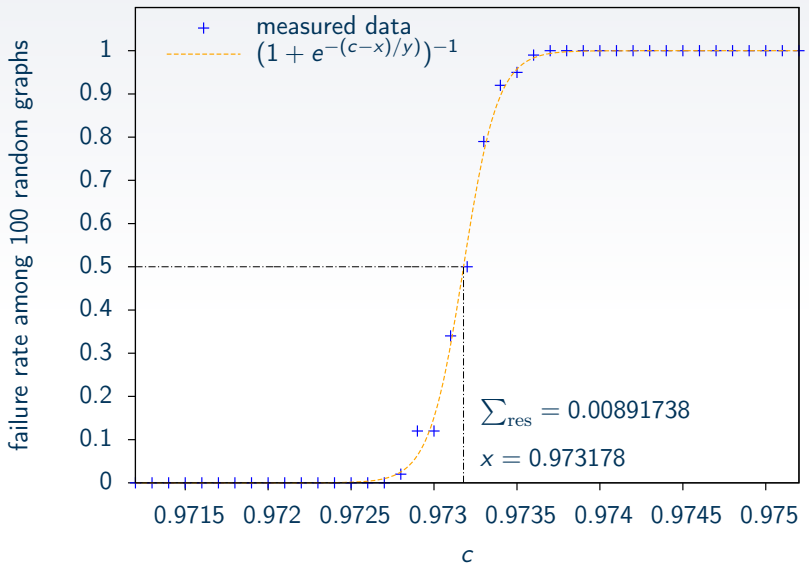


3 primary, 1 backup choice



←– interval grows –→

⟵ interval shift

Threshold: **decreasing** **stable (≈ 4-ary c.h.)**

page size	4 choices, 1 page (no backup)	3 primary, 1 backup choice
10^5	0.972	0.977
10^4	0.952	0.977
10^3	0.879	0.977
10^2	0.649	0.977
10^1	0.188	0.973

Primary Keys

From now on **only paging with backup option**.

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0.95	0.872	0.974	0.976

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- increases with growing page size
- decreases with increasing load
- load of primary keys (=fraction · load) can **exceed** threshold for 3-ary c.h. (≈ 0.918)

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example: $c = 0.95$, $s = 10^3$, fraction of primary keys 0.974

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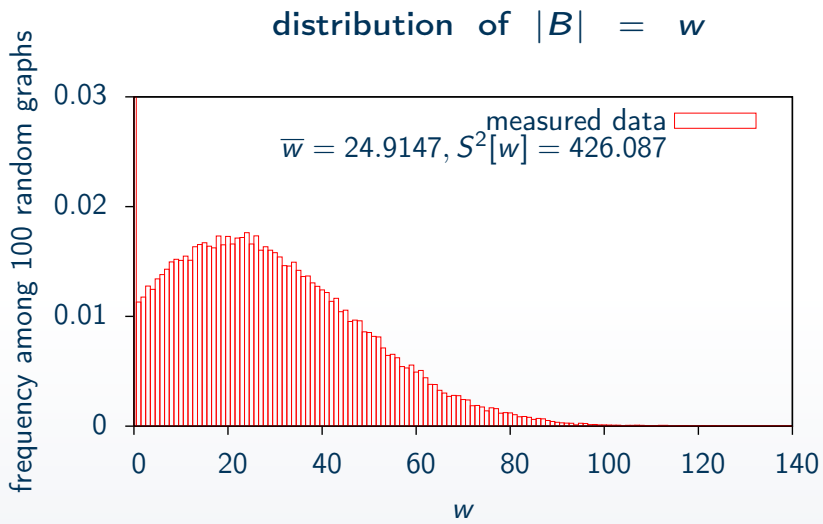

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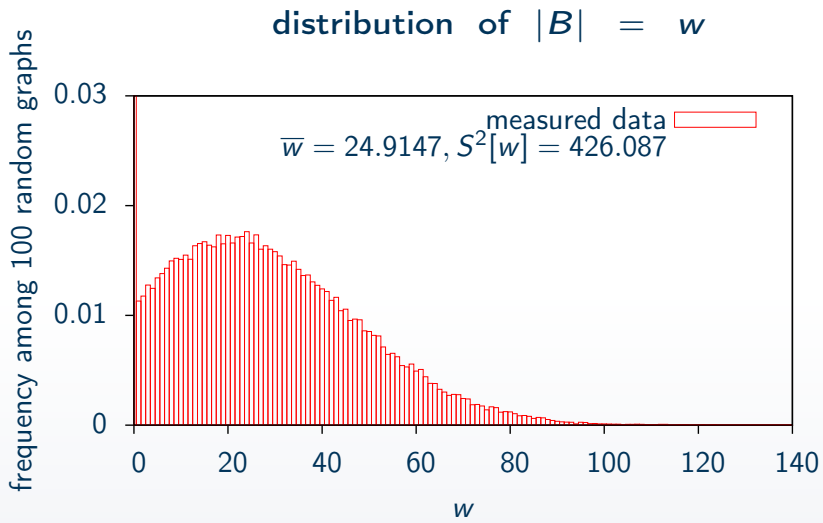

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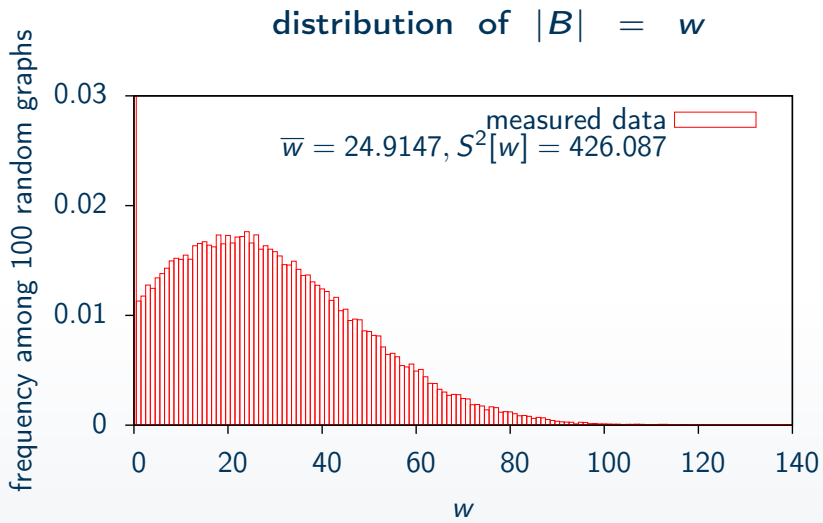
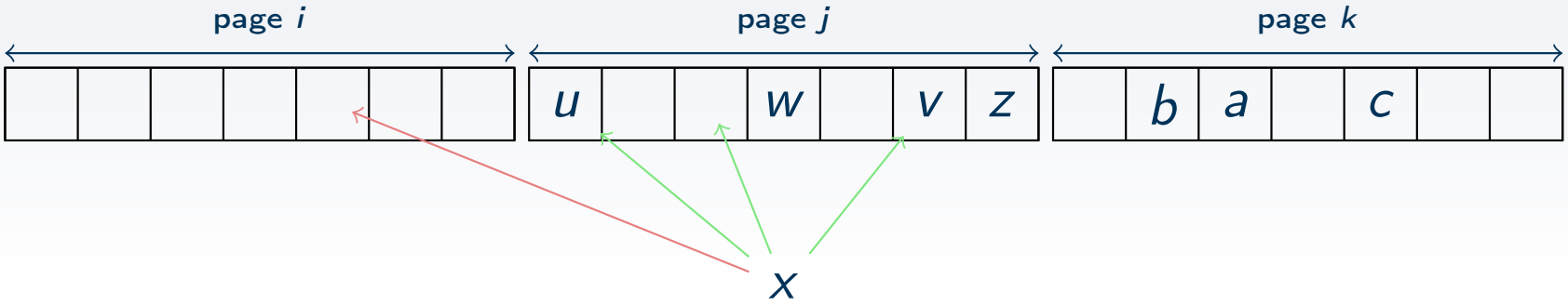


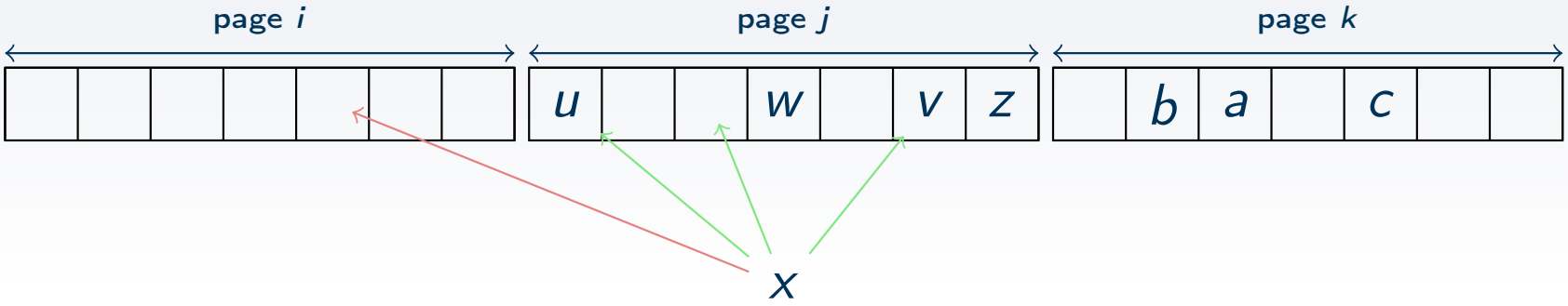
$\Rightarrow \mathrm{E}(X_u) < 1.0015$

Dynamic Case



Simple Random-Walk:

Dynamic Case



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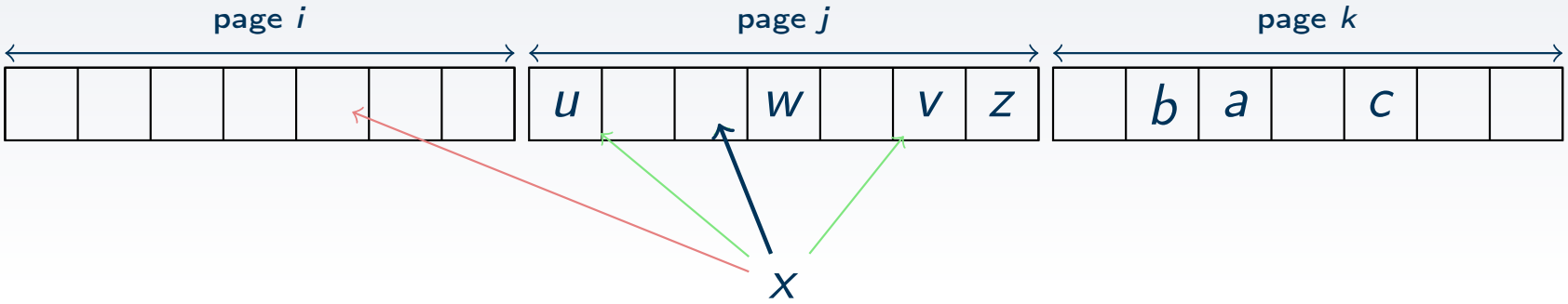

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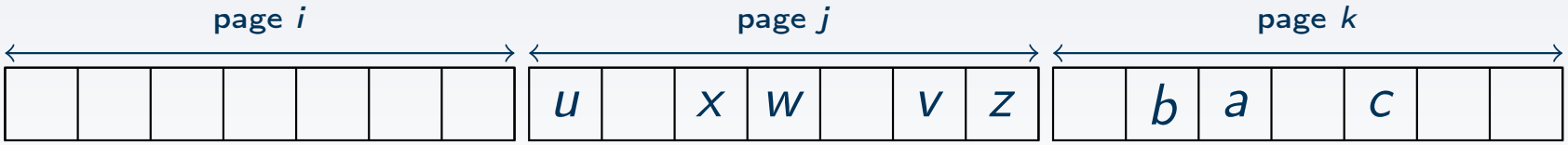

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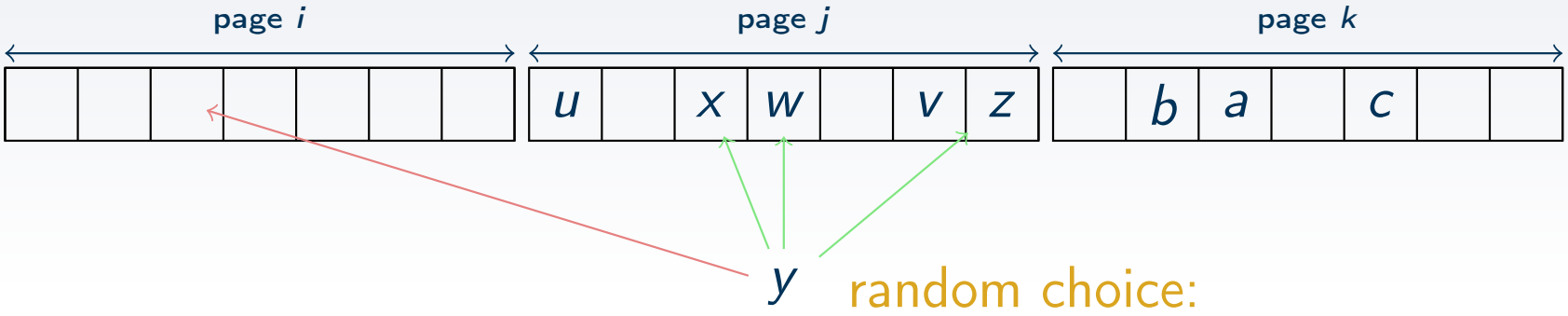

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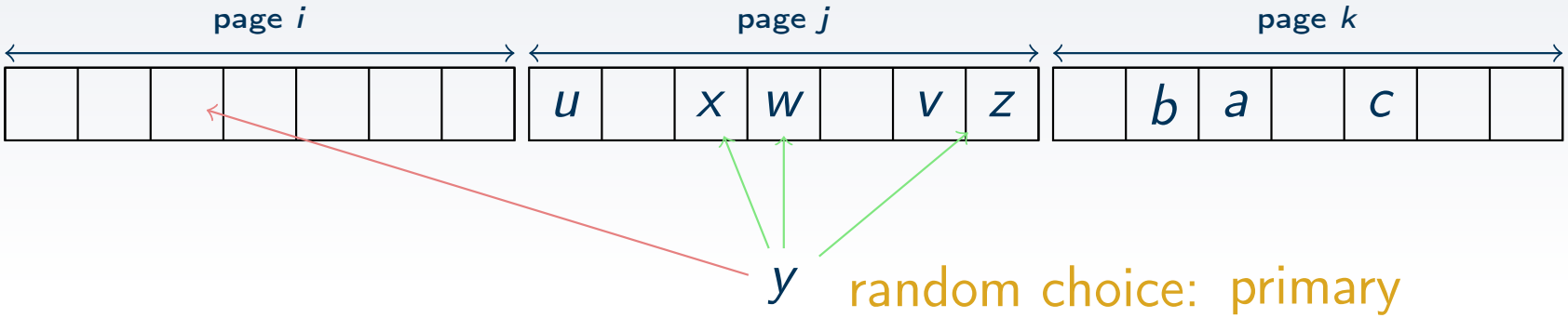

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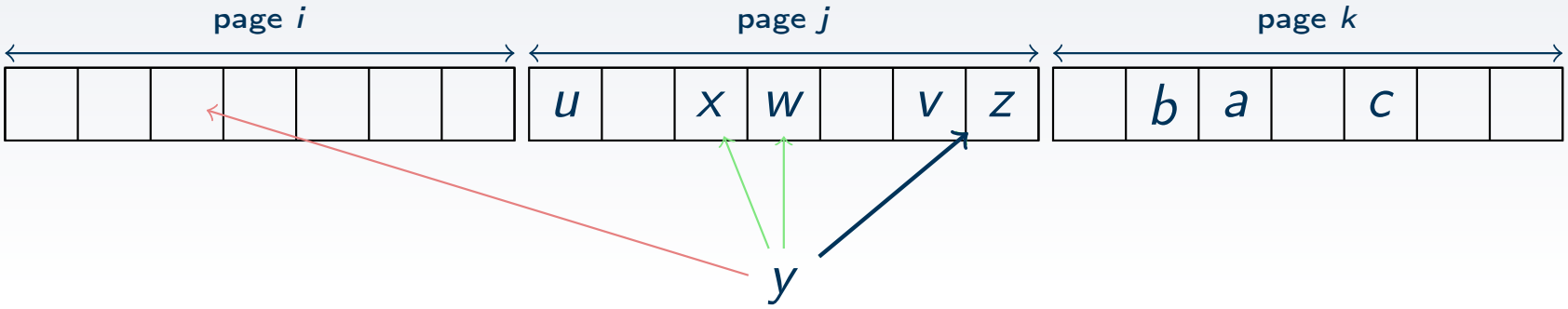

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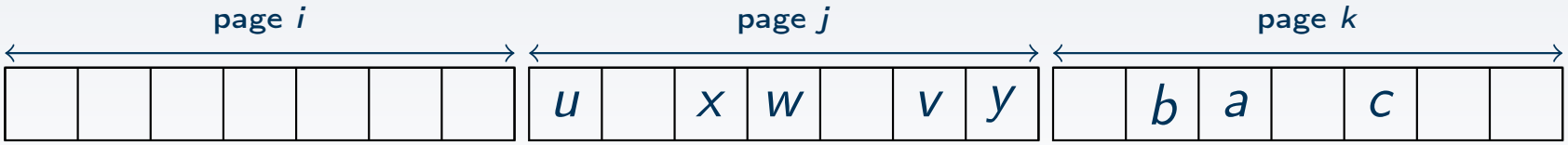

z

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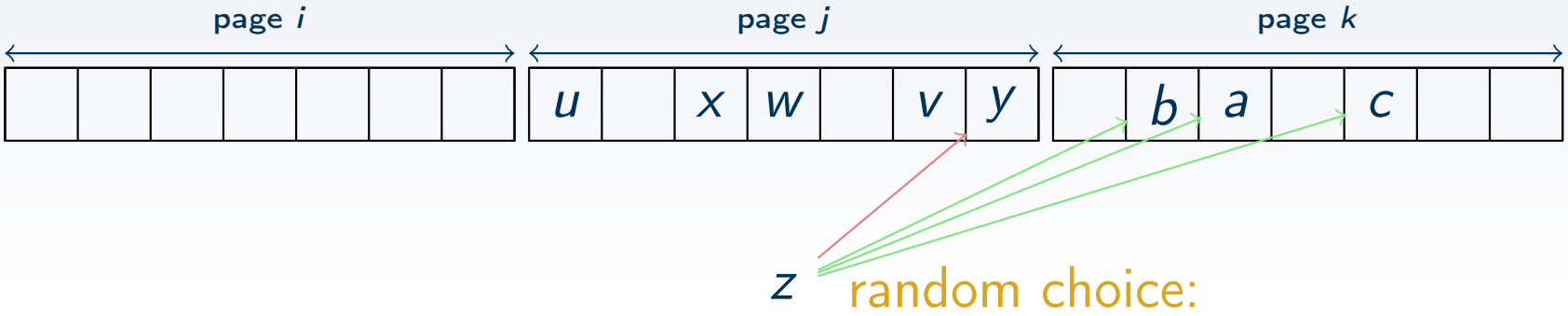

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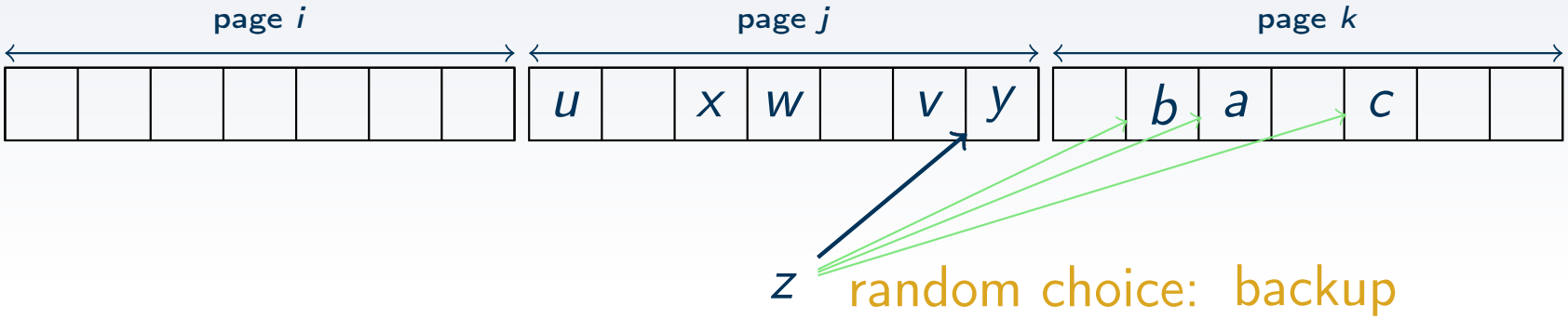

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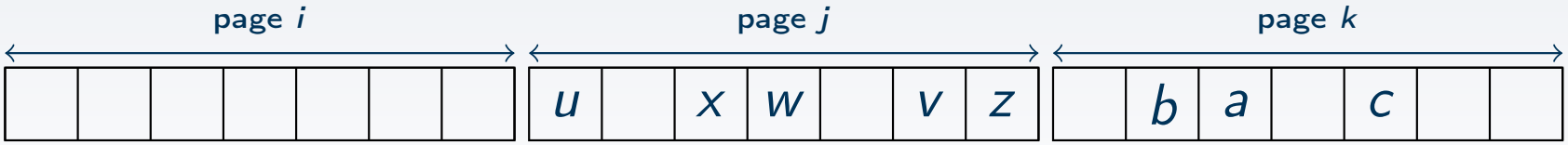

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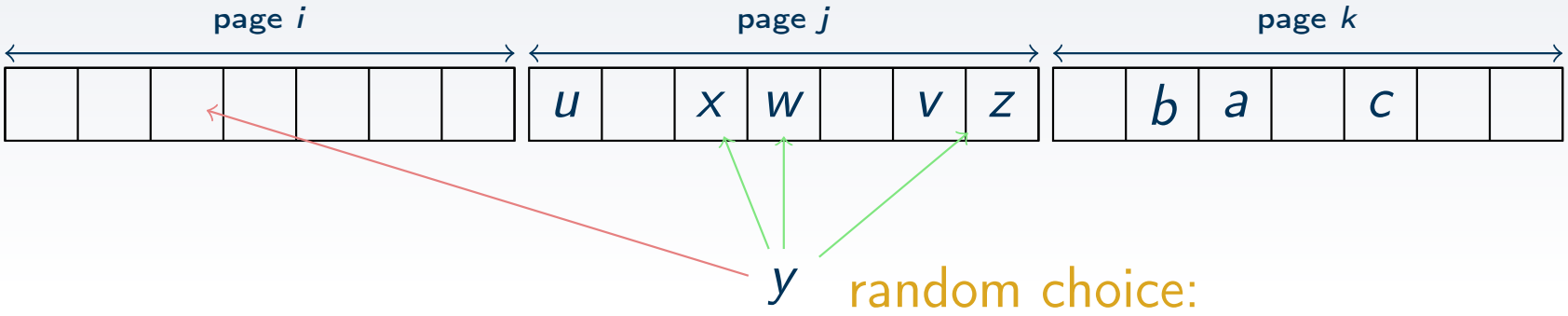

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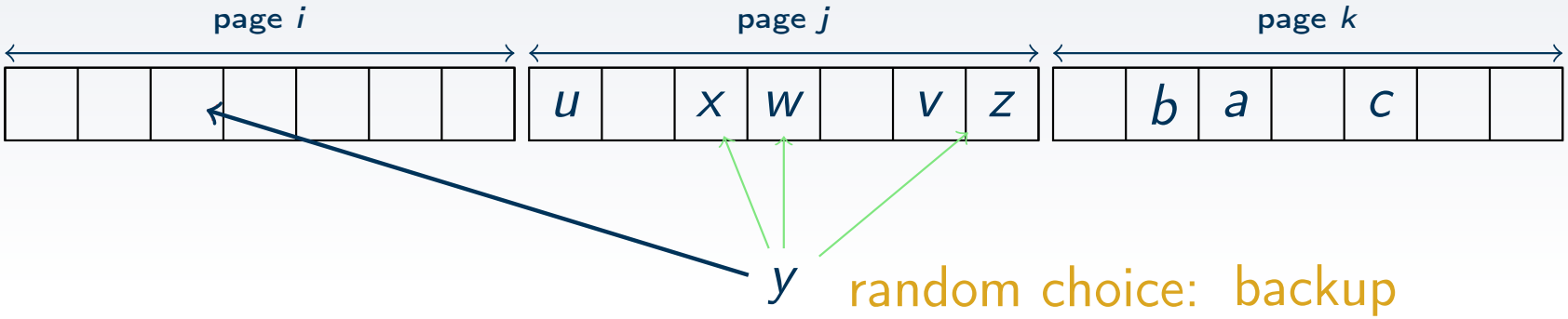

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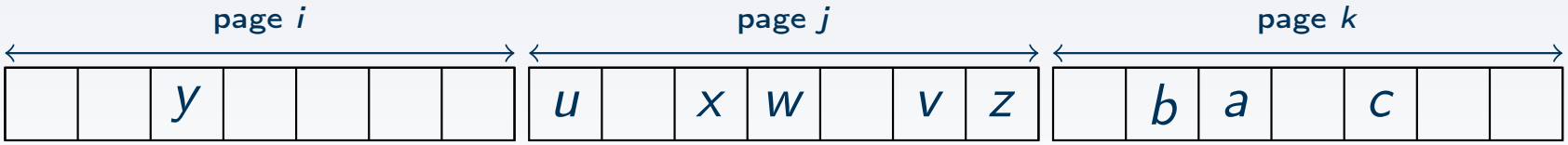

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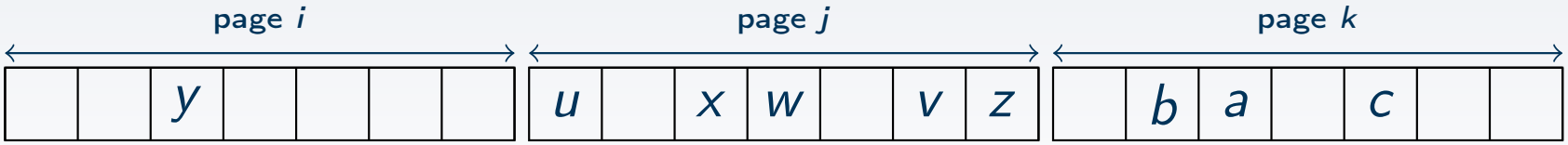

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- Parameter: bias, e.g. choose primary page with probability 0.95

Dynamic vs Static (1)

Fraction of primary keys:

page size:		10^2	10^3
bias	load		
0.97	0.95		
0.90	0.97		

Dynamic vs Static (1)

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page size:		10^2	10^3
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0.97	0.95	0.938 [0.955]	0.956 [0.974]
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- missed maximum achievable fraction [static case] up to 0.02 **for load** $c = 0.95$

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0.90	0.97	0.887 [0.957]	0.898 [0.936]

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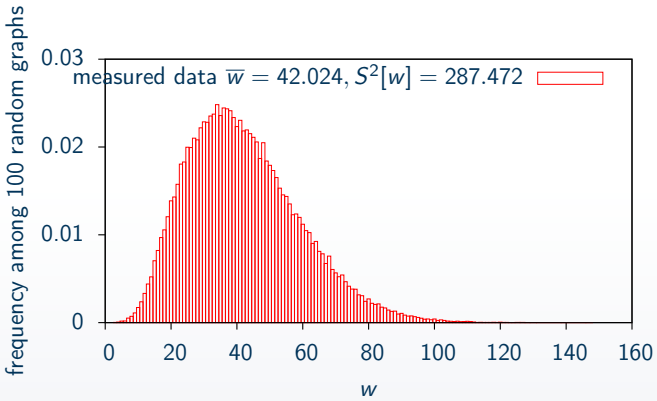

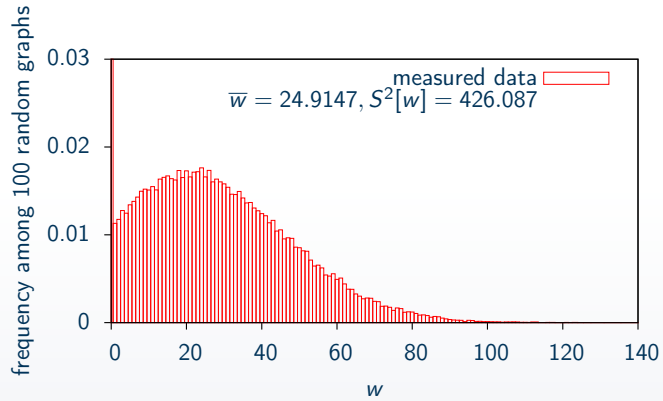

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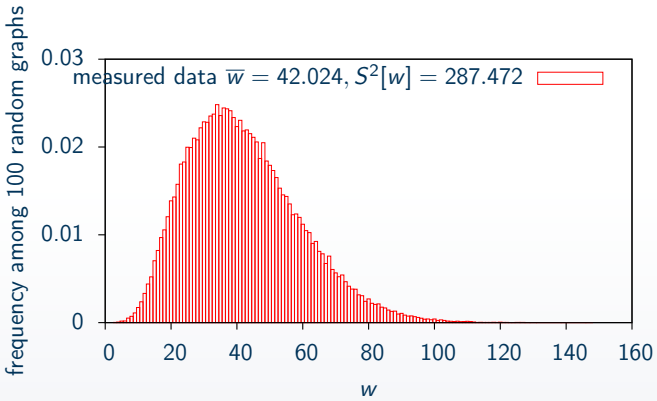

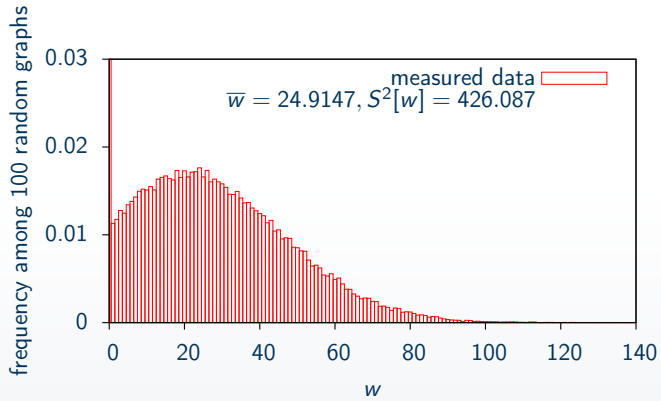

- dynamic $\mathrm{E}(X_u) = 1.0043$, static $\mathrm{E}(X_u) = 1.0015$

Summary

Cuckoo hashing with pages - 3 primary, 1 backup option:

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More results and details, e.g. concerning **performance of the random walk algorithm**, **deletions** and very **small pages**, see ESA paper and full version on arXiv.

Open Problem

Provable performance bounds
for cuckoo hashing with pages

e.g. threshold behavior in offline szenario

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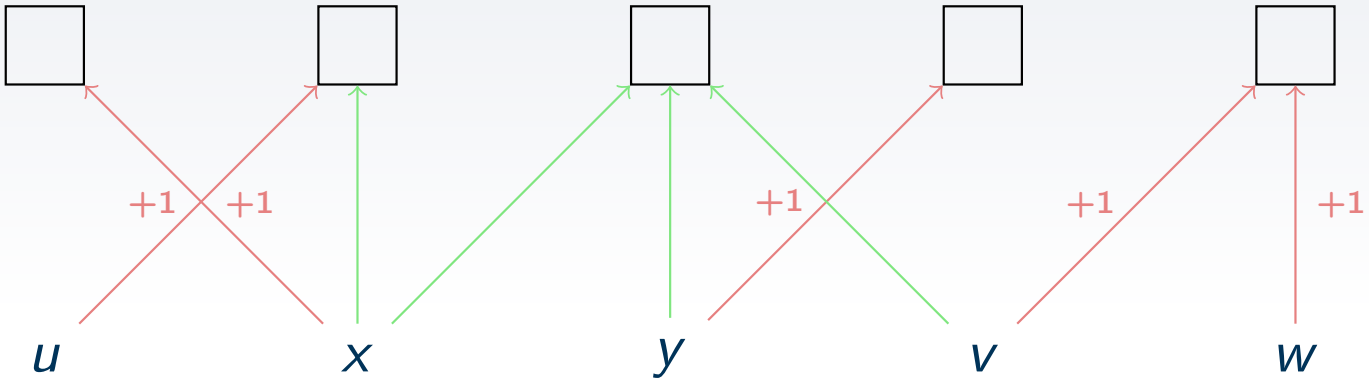
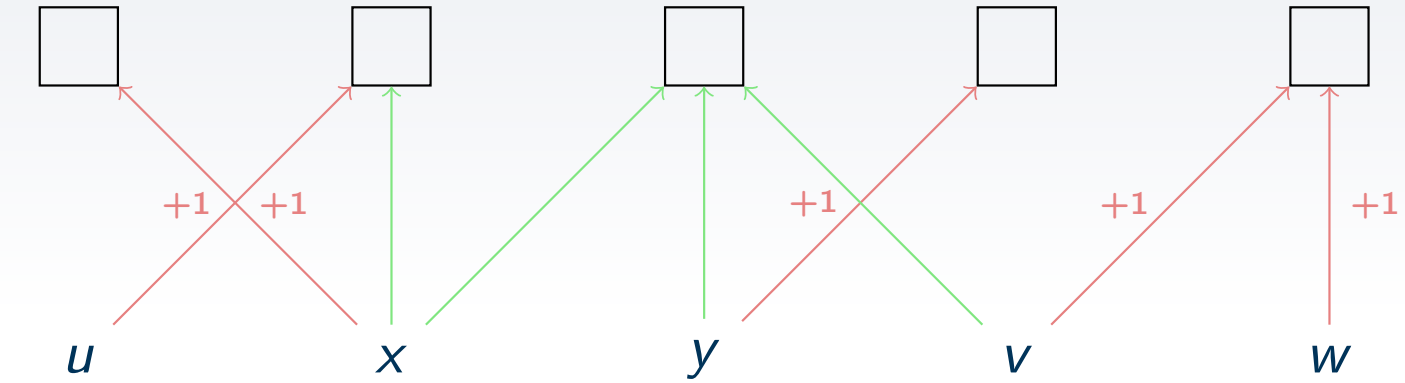
Successive Shortest Path Algorithm



Variant of Hopcroft-Karp algorithm:

- primary edges have cost 0, backup edges have cost $+1$ or -1

Successive Shortest Path Algorithm



$C = 0$

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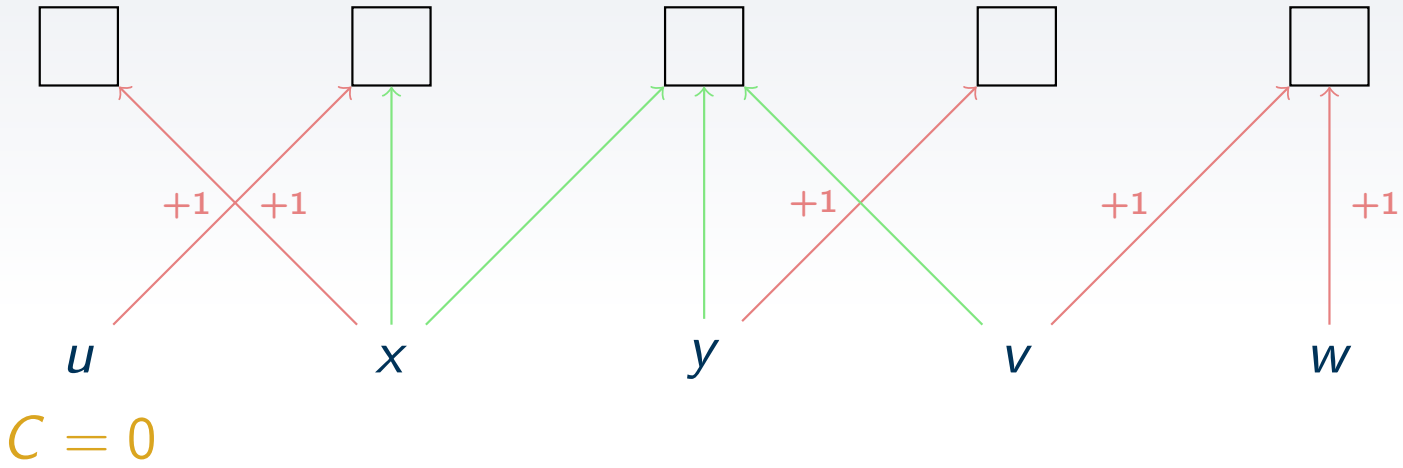

Variant of Hopcroft-Karp algorithm:

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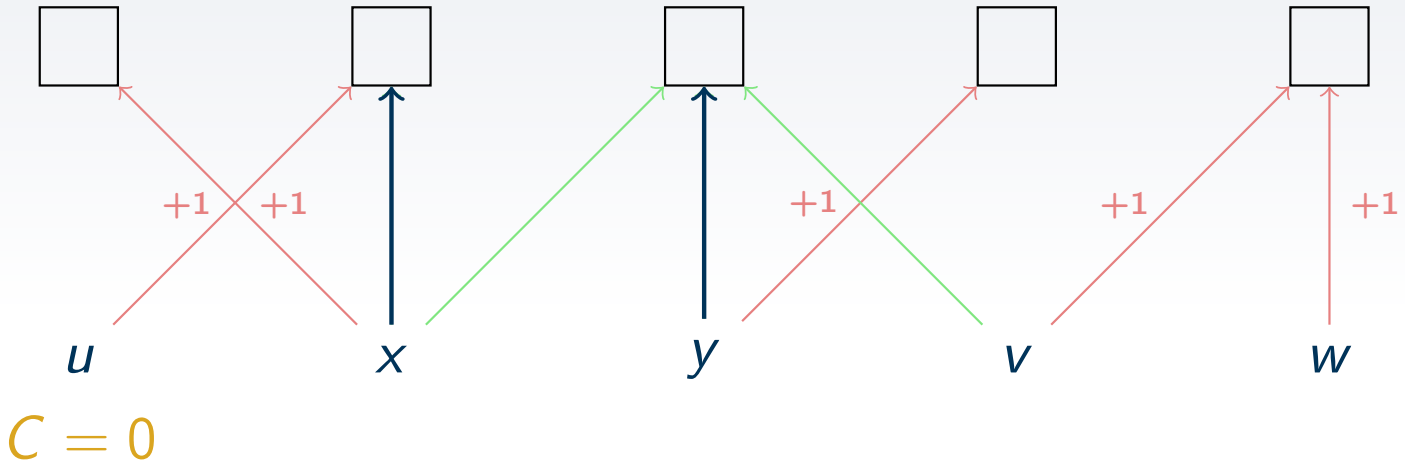
- search for augmenting path of fixed costs C, start with $C = 0$
- as long as there is any augmenting path

Successive Shortest Path Algorithm



Variant of Hopcroft-Karp algorithm:

- primary edges have cost 0, backup edges have cost $+1$ or -1
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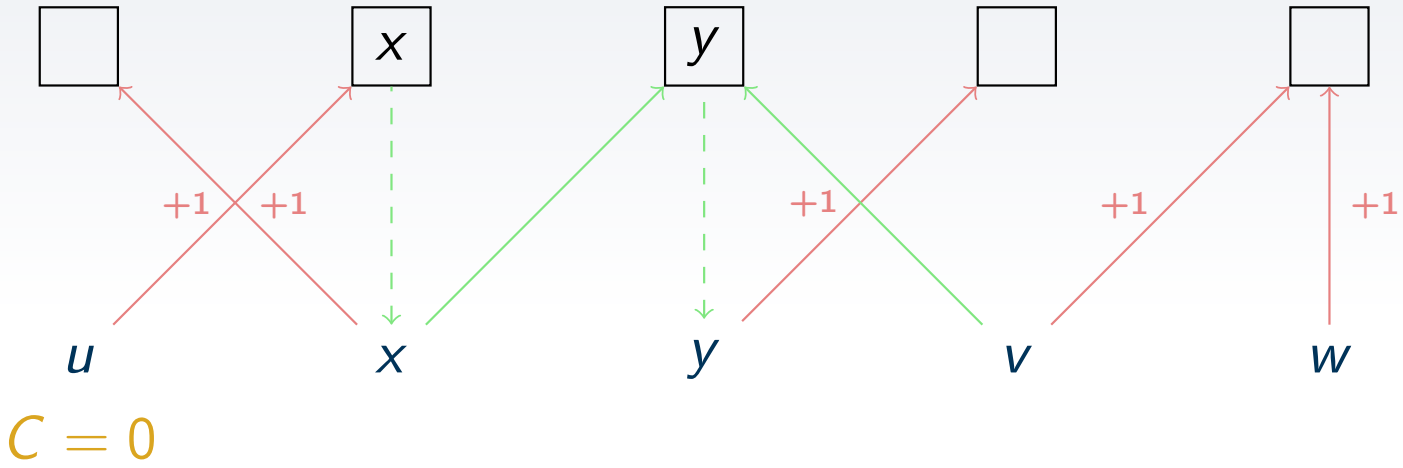

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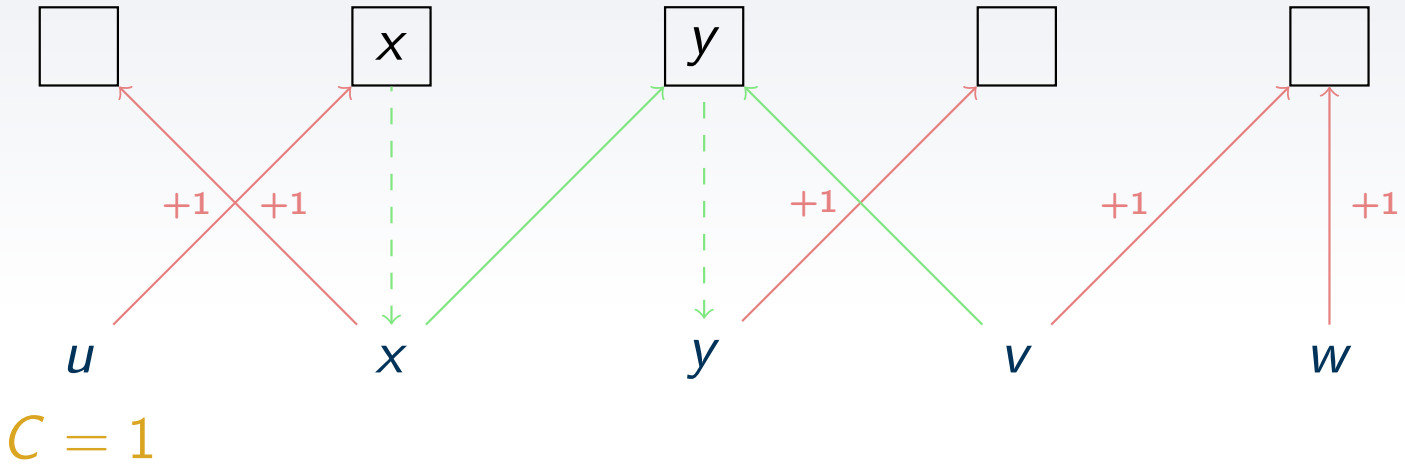

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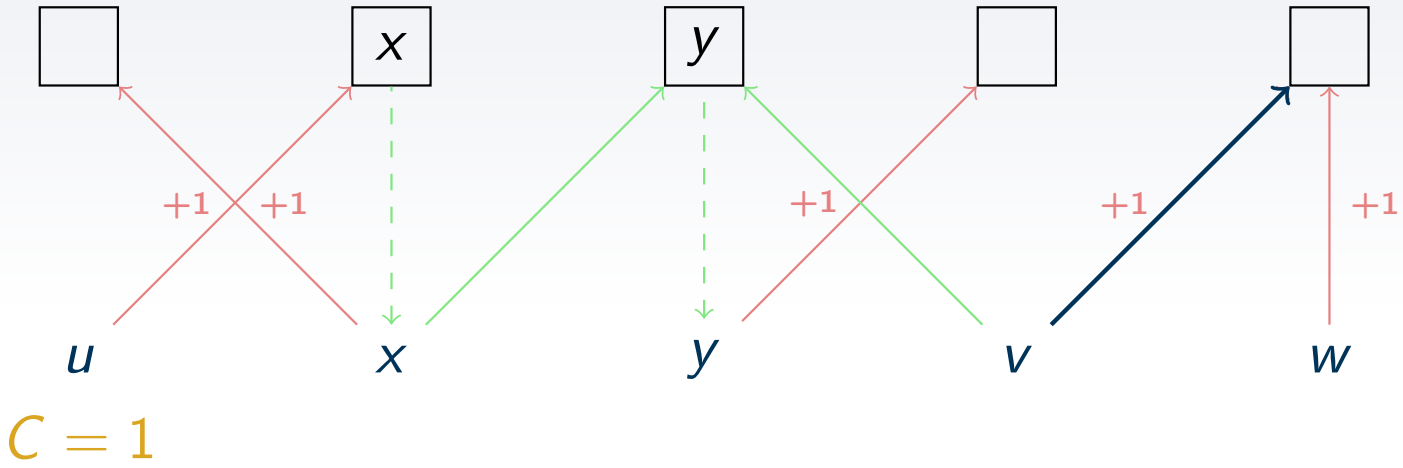
Successive Shortest Path Algorithm



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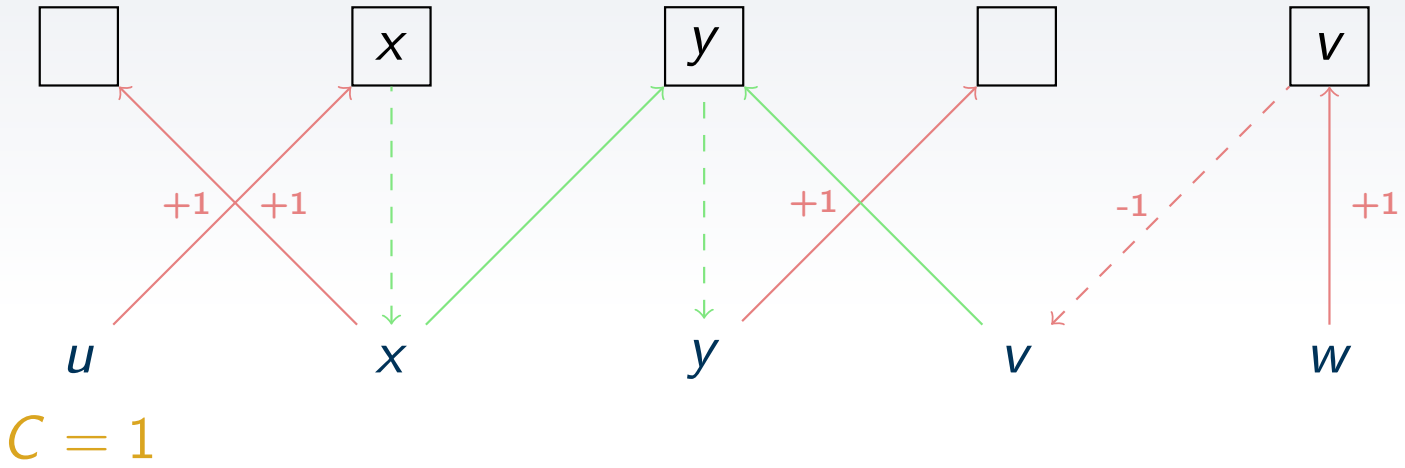

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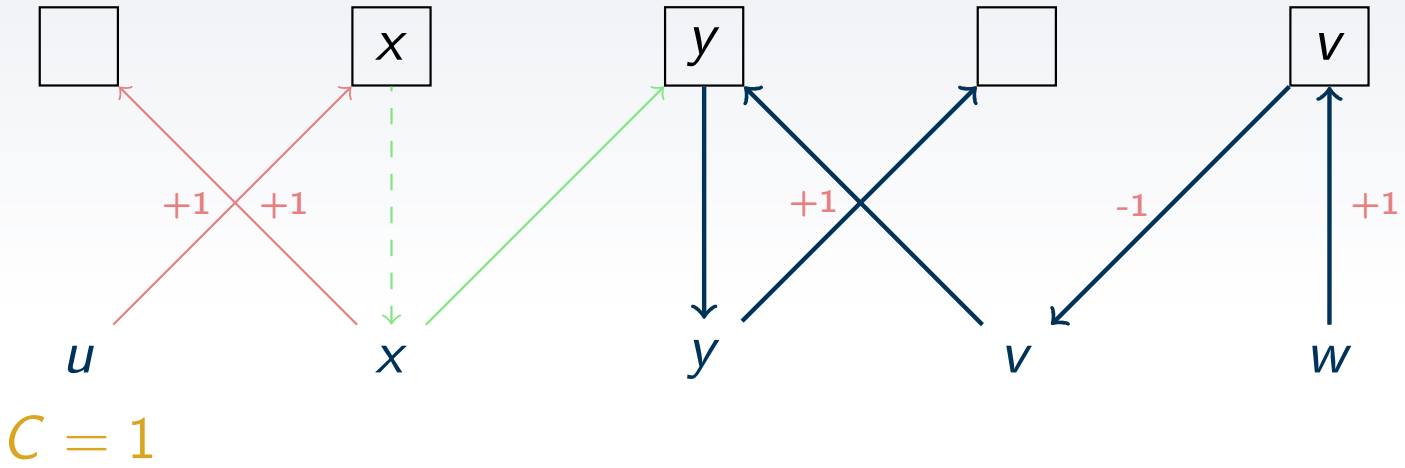

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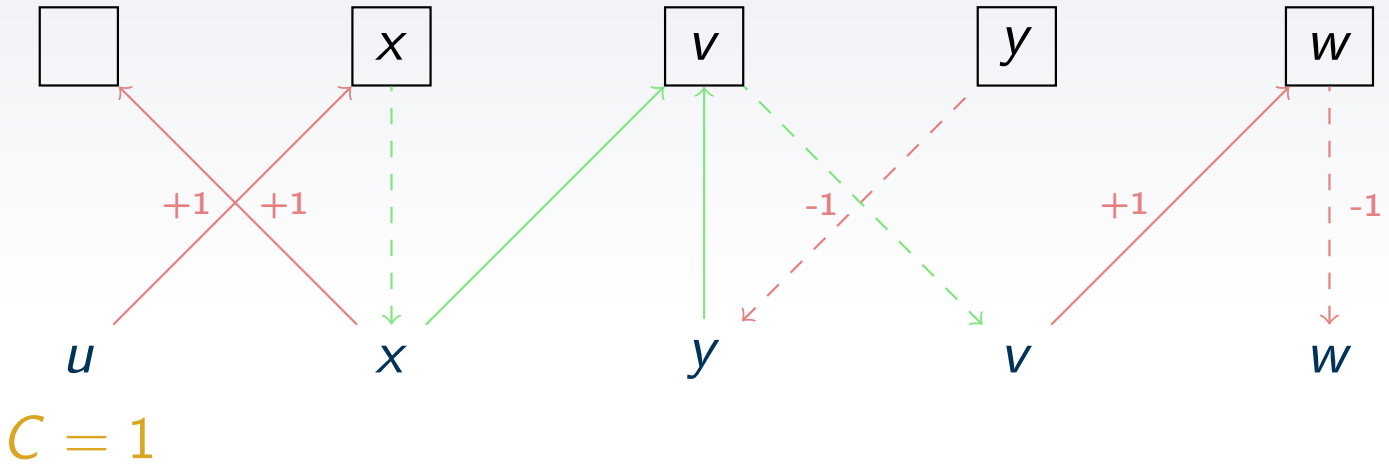

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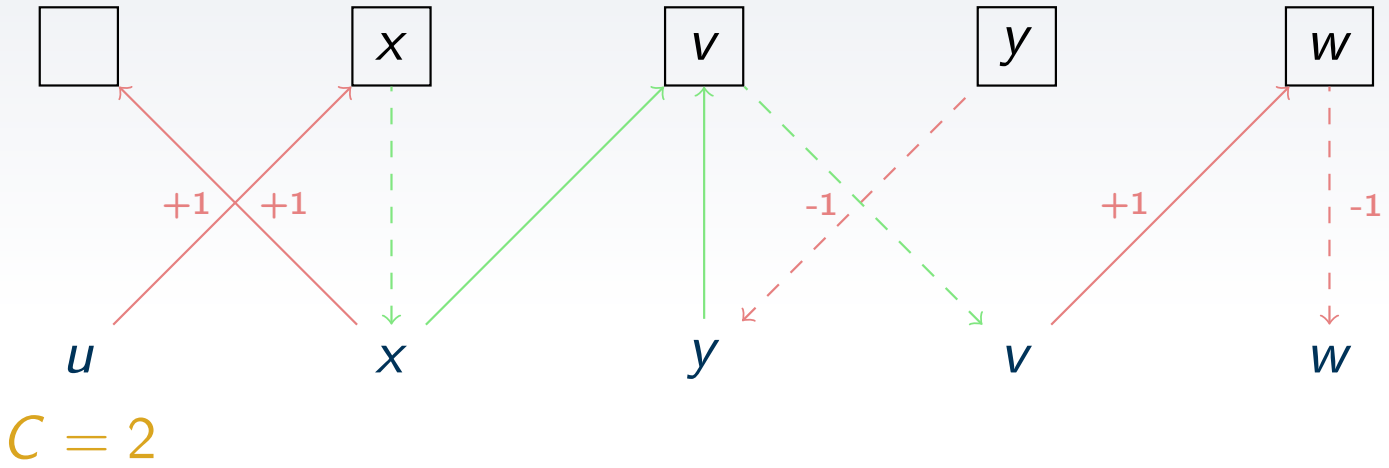

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Variant of Hopcroft-Karp algorithm:

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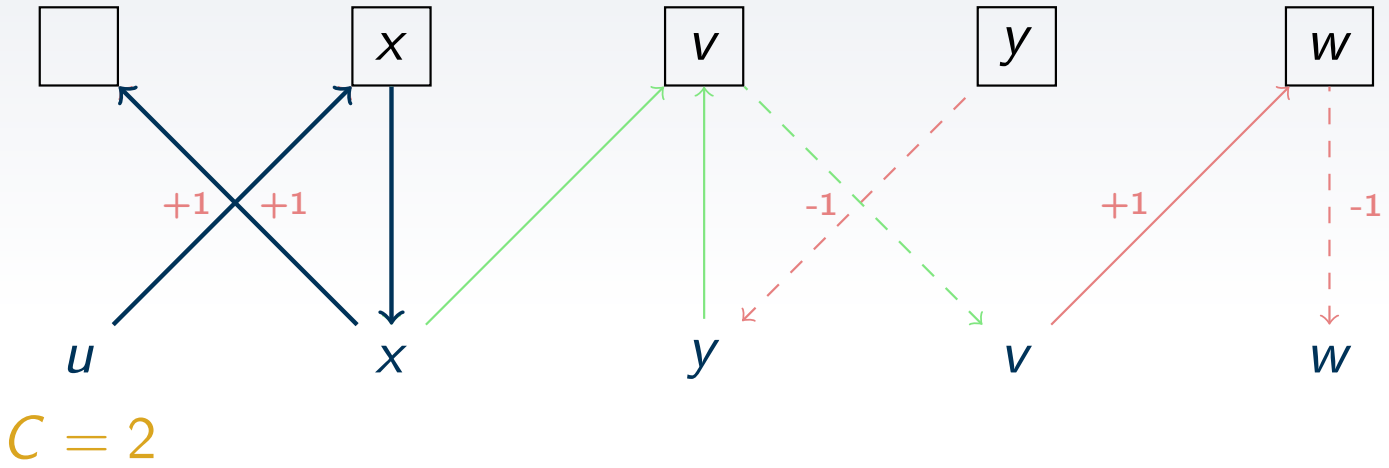
Successive Shortest Path Algorithm



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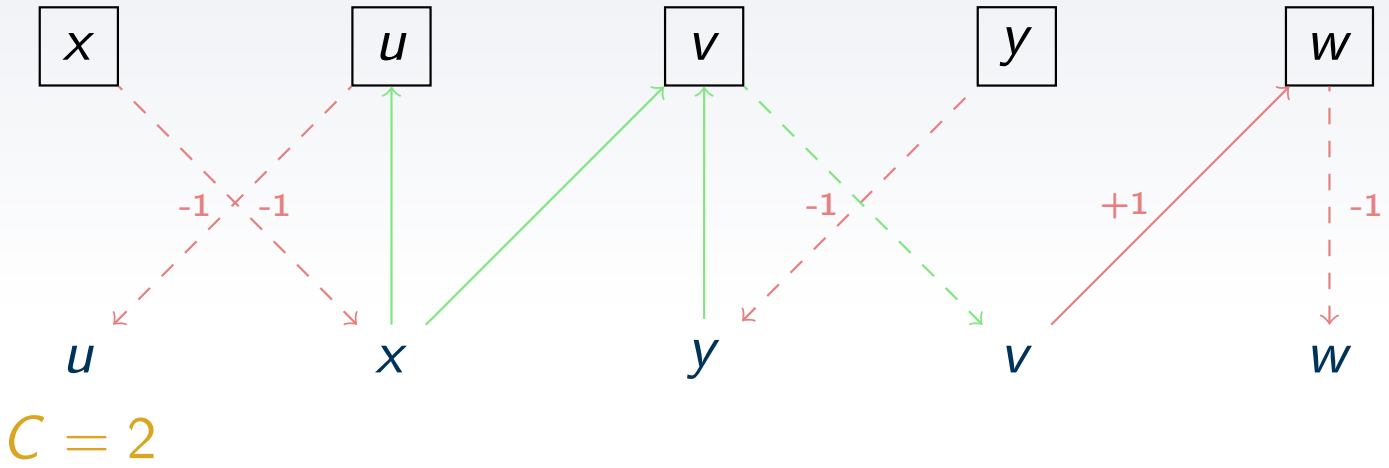
Successive Shortest Path Algorithm



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Properties of Random Walk (1)

Failure rate among 10^6 attempts:

max avg. #steps:		30	25
page size:		10^2	10^3
bias	load		
0.97	0.95		
0.90	0.97		

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$\Rightarrow$ **failure probability is** $< 10^{-5}$ with level of significance $> 1 - e^{-10}$

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bias	load		
0.97	0.95	0	0
0.90	0.97	0	0

$\Rightarrow$ **failure probability is** $< 10^{-5}$ with level of significance $> 1 - e^{-10}$

$\Rightarrow$ ignore max avg. #steps for these configurations

Properties of Random Walk (2)

Average number of steps and page requests:

page size:		10^2		10^3	
bias	load				
0.97	0.95	(	)	(	)
0.90	0.97	(	)	(	)

Properties of Random Walk (2)

Average number of steps and page requests:

page size:		10^2	10^3
bias	load		
0.97	0.95	(22.81, 2.25)	( )
0.90	0.97	( )	( )

Properties of Random Walk (2)

Average number of steps and page requests:

page size:		10^2	10^3
bias	load		
0.97	0.95	(22.81, 2.25)	(16.60, 1.89)
0.90	0.97	( )	( )

- decreasing for growing page size

Properties of Random Walk (2)

Average number of steps and page requests:

page size:		10^2	10^3
bias	load		
0.97	0.95	(22.81, 2.25)	(16.60, 1.89)
0.90	0.97	(23.29,)	(19.49,)

- decreasing for growing page size
- **moderate number of steps**

Properties of Random Walk (2)

Average number of steps and page requests:

page size:		10^2	10^3
bias	load		
0.97	0.95	(22.81, 2.25)	(16.60, 1.89)
0.90	0.97	(23.29, 5.36)	(19.49, 4.61)

- decreasing for growing page size
- **moderate number of steps**
- number of page requests high near maximum load threshold